Wealth-driven asymptotic survival in a financial market with demand shocks

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This version: 14th June 2017

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Outline

Context and motivation

Friedman *'as if'* argument (1953)

- economic agents can be described *as if* they were fully rational
- non-rational agents would get wiped out of the market

Empirical evidence

- trade occurs
- excess volatility
- market sentiment

Heterogeneous Agent Models (HAMs)

- analytical investigation of long-run dynamics
- incorporate various degrees of bounded rationality

Context and motivation

traditional HAMs

- feedback mechanisms from realised market outcomes
  - e.g. fundamentalist vs. chartist
- long-run analysis within a notion of equilibrium
  - i.e. actions are fixed at the equilibrium
- *'deterministic skeleton'* approach

what happens if one introduces *persistent* demand shocks?

Black (1986) JoF

noise traders: *"trading on noise as if it were information"*

Noise traders vs. rational arbitrageurs

De Long et al. (1990) JPE price impact, fixed wealth

De Long et al. (1991) JoB endogenous wealth, no price impact

Context and motivation

Our contribution

- constant vs. stochastic portfolio strategies
- wealth-driven market selection
- conditions for survival and dominance

Main findings

- trade-off between portfolio riskiness and variability
- long-run heterogeneity
- non-trivial price-wealth dynamics (e.g. volatility clustering)
- loss of generality in adopting the deterministic skeleton approach

Outline

The model

Consider a stylised financial market (no consumption)

- trade takes place in discrete time t
- risk-free bond
  - perfectly elastic supply
  - constant r_f return
  - price normalised to 1 (*numéraire*)
- long-lived risky security
  - positive dividend $d_t = d_{t-1}(1+g)$, $g > r_f$
  - unitary constant supply
  - market clearing price p_t

At each time step t trader n

- invests a fraction $x_{n,t}$ of her wealth $w_{n,t}$ into the risky security
- residually invests $(1 - x_{n,t}) \cdot w_{n,t}$ into the bond

Trader n wealth $w_{n,t}$ equals the current market value of her portfolio

Traders' behaviour

Assumption

- constant trader always invests $\overline{x} \in (0,1)$
- stochastic trader invests according to Markov process $\{\mathcal{X}_t,\ t \in \mathbb{N}\}$

$\pi^u > 0$

$\pi^d > 0$

$0 < x^d < x^u < 1$

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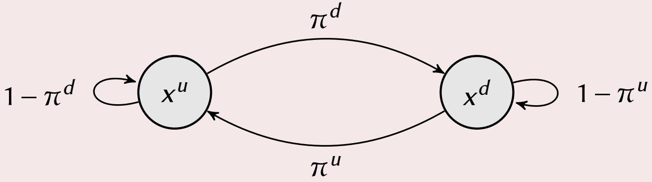

$$\pi^u > 0 \qquad \pi^d > 0 \qquad 0 < x^d < x^u < 1$$

Laws of motion of the economy

$$\mathcal{F}_{x_{t-1},x_t} : \mathcal{D} \to \mathcal{D}, \qquad \mathcal{D} = \Delta \times (-1, +\infty) \times \mathbb{R}_{++}$$

$$\begin{cases} \varphi_t = \varphi_{t-1} \dfrac{1 + x_{t-1}(r_t + e_t)}{1 + (r_t + e_t)\left[\varphi_{t-1} x_{t-1} + (1 - \varphi_{t-1})\overline{x}\right]} \\ r_t = \dfrac{\varphi_{t-1}\left[x_t(1 + e_t x_{t-1}) - x_{t-1}\right] + (1 - \varphi_{t-1}) e_t \overline{x}^2}{\varphi_{t-1} x_{t-1}(1 - x_t) + (1 - \varphi_{t-1})\overline{x}(1 - \overline{x})} \\ e_t = e_{t-1} \dfrac{1 + g}{1 + r_{t-1}} \end{cases}$$

where

$$\varphi_t = \frac{\text{wealth of stochastic trader}}{\text{total wealth}} \in [0, 1]$$

Outline

Long-run outcomes

Terminology

- trader n is said to *survive* on $\{x_t\}_{t=0}^{\infty}$ if $\limsup_{t\to\infty} \varphi_{n,t} > 0$
- trader n is said to *vanish* on $\{x_t\}_{t=0}^{\infty}$ if $\limsup_{t\to\infty} \varphi_{n,t} = 0$
- trader n is said to *dominate* on $\{x_t\}_{t=0}^{\infty}$ if $\liminf_{t\to\infty} \varphi_{n,t} = 1$

Proposition

If $(\varphi^*, r^*, e^*) \in \mathcal{D}$ is a fixed point of system $\mathcal{F}$ then either

1. the constant trader dominates (steady state $\mathfrak{C}$)
2. the stochastic trader dominates (steady state $\mathfrak{S}$)

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A dominant constant trader

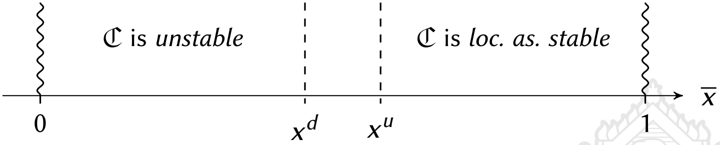
$$\varphi^{\mathfrak{C}} = 0 \qquad r^{\mathfrak{C}} = g \qquad e^{\mathfrak{C}} = g\frac{1-\overline{x}}{\overline{x}}$$

Proposition

Steady state $\mathfrak{C}$ is locally asymptotically stable if

$$\lambda^{\mathfrak{C}} = \frac{\left(\overline{x} + g x^{u}\right)^{\frac{\pi^{u}}{\pi^{u}+\pi^{d}}} \left(\overline{x} + g x^{d}\right)^{\frac{\pi^{d}}{\pi^{u}+\pi^{d}}}}{\overline{x}(1+g)} < 1$$

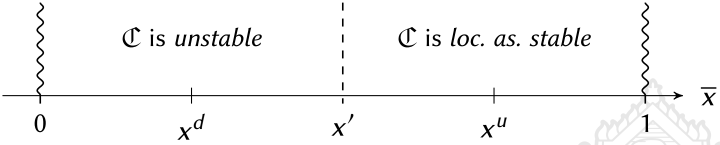
A dominant constant trader



Sufficient conditions

- $\bar{x} \geq x^u \implies \mathfrak{C}$ is locally asymptotically stable
- $\bar{x} \leq x^d \implies \mathfrak{C}$ is unstable

A dominant constant trader



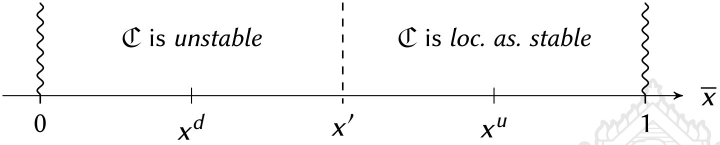
Proposition

$\exists!\ x' \in (0,1)$ such that

- $\forall\ \overline{x} > x'$, $\mathfrak{C}$ is locally asymptotically stable
- $\forall\ \overline{x} < x'$, $\mathfrak{C}$ is unstable

Moreover $x^d < x' < x^u$

A dominant constant trader



Special case

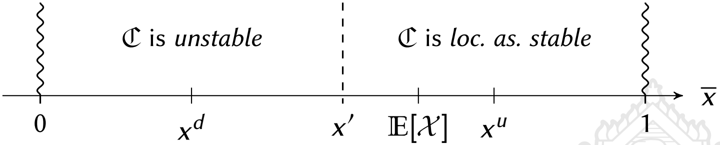
$$\pi^u = \pi^d \quad \Longrightarrow \quad x' = \frac{x^u + x^d}{2} - h(g)$$

$$h(0) = 0$$
$$h(g) > 0$$
$$h'(g) > 0$$

Proposition

$x' < \mathbb{E}[\mathcal{X}]$ even if $\pi^u \neq \pi^d$

A dominant constant trader



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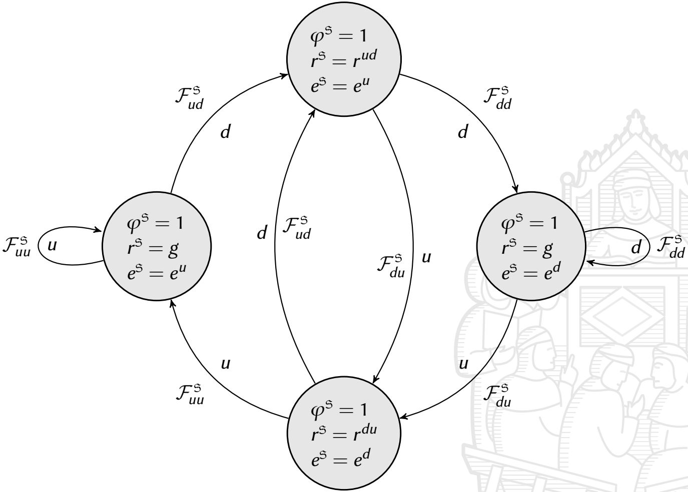
A dominant stochastic trader



It holds $r^{ud} < g < r^{du}$ and $e^{u} < e^{d}$. Moreover, *'usually'* $r^{ud} < 0$

A dominant stochastic trader

Proposition

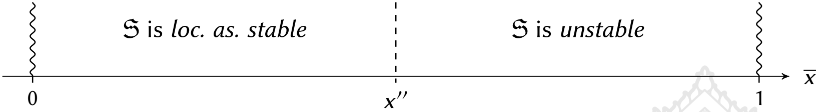
Steady state $\mathfrak{S}$ is locally asymptotically stable if

$$\lambda^{\mathfrak{S}} = \frac{1}{1+g}\left[1+\frac{g\overline{x}}{x^u}\right]^{\frac{\pi^u(1-\pi^d)}{\pi^u+\pi^d}}$$

$$\cdot\left[\frac{\overline{x}\Big[g(1-x^u)-(x^u-x^d)\Big]+x^u(1-x^d)}{x^u(1-x^u)}\cdot\frac{\overline{x}\Big[g(1-x^d)+(x^u-x^d)\Big]+x^d(1-x^u)}{x^d(1-x^d)}\right]^{\frac{\pi^u\pi^d}{\pi^u+\pi^d}}$$

$$\cdot\left[1+\frac{g\overline{x}}{x^d}\right]^{\frac{\pi^d(1-\pi^u)}{\pi^u+\pi^d}} < 1$$

A dominant stochastic trader

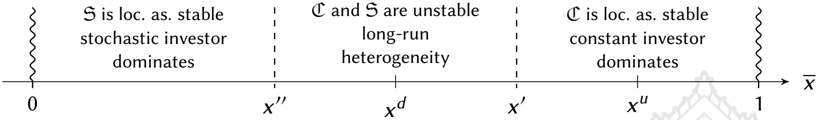


Numerical result

$\exists!\ x'' \in (0, 1)$ such that

- $\forall\ \overline{x} < x''$, $\mathfrak{S}$ is locally asymptotically stable
- $\forall\ \overline{x} > x''$, $\mathfrak{S}$ is unstable

Long-run heterogeneity

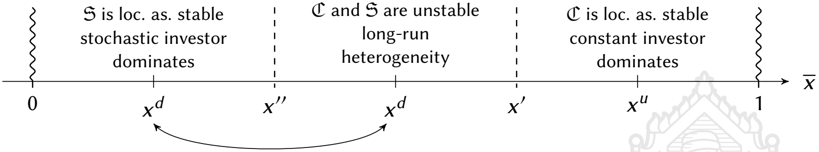


Proposition

$\exists\ \hat{g} > 0$ such that $\forall\ g < \hat{g}$ it holds $x'' < x^d < x'$. In particular

$$\hat{g} = \frac{x^u - x^d}{1 - x^u}$$

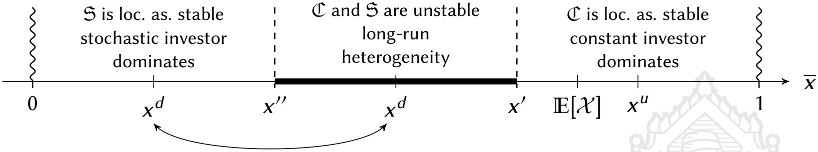
Long-run heterogeneity



Numerical result

the relation $x'' < x'$ holds

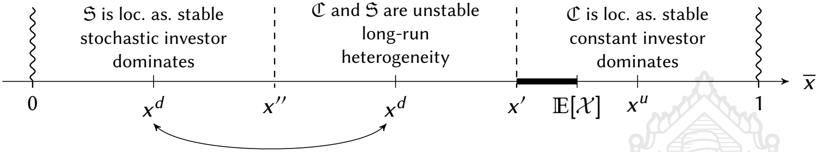
- $\forall\ x^u, x^d, \pi^u, \pi^d \in \{0.01, 0.02, \dots, 0.99\}$ such that $x^d < x^u$
- $\forall\ g = \hat{g} \cdot 10^k, k \in \mathbb{N}_+$ such that $g \leq 10$



Corollary

1. there exists a non-degenerate interval (x'', x') such that both $\mathfrak{C}$ and $\mathfrak{S}$ are unstable $\forall\ \overline{x} \in (x'', x')$
2. the constant trader is able to *dominate* the stochastic trader even adopting a on-average safer position $\overline{x} < \mathbb{E}[\mathcal{X}]$
3. the constant trader is able to *invade* the stochastic trader even adopting an always strictly safer position $\overline{x} < x^d < x^u$ when $x'' < x^d$

Long-run heterogeneity



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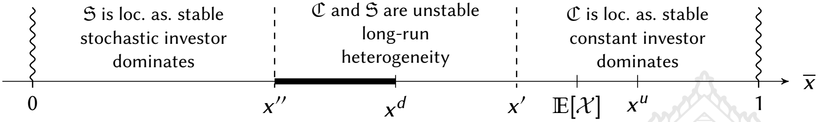

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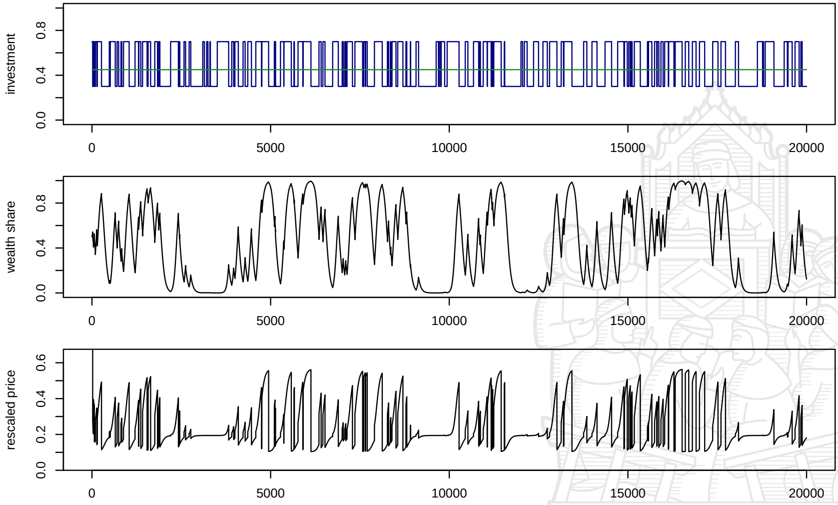
Outline

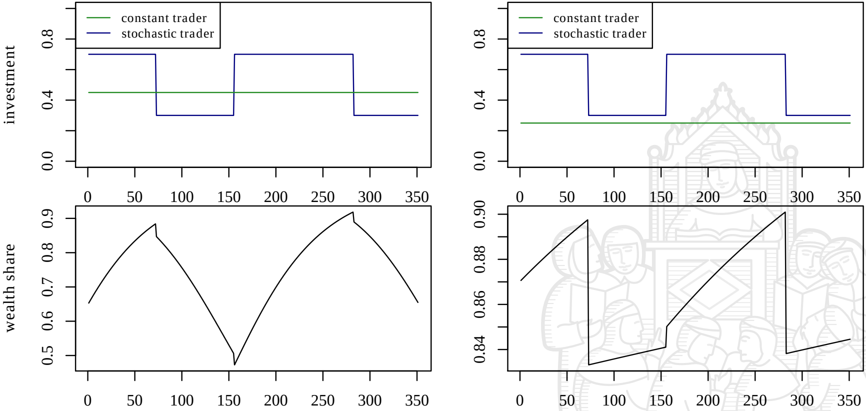
Initialisation

Description	Variable	Value
stochastic investment up	x^u	0.7
stochastic investment down	x^d	0.3
probability down when up	π^d	0.01
probability up when down	π^u	0.01
constant investment	$\overline{x}$	0.45
dividend rate of growth	g	0.05
initial wealth share	φ_0	0.5
initial return	r_0	0.0
initial yield	e_0	0.01

Simulation


investment
0.0
0.4
0.8
0
5000
10000
15000
20000
wealth share
0.0
0.4
0.8
0
5000
10000
15000
20000
rescaled price
0.0
0.2
0.4
0.6
0
5000
10000
15000
20000

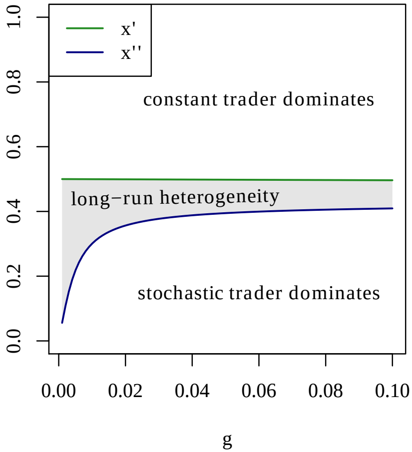
A tale of two regimes

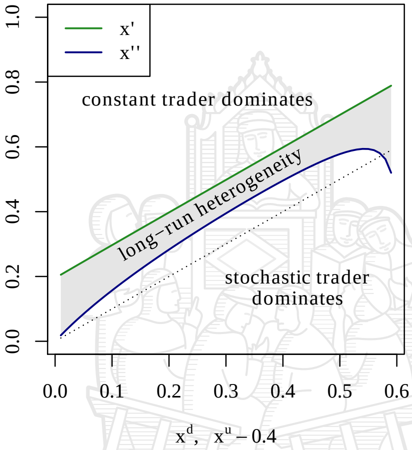


1st regime: $g = 0.05$ and $\overline{x} = 0.45$

2nd regime: $g = 0.005$ and $\overline{x} = 0.25$

Sensitivity analysis





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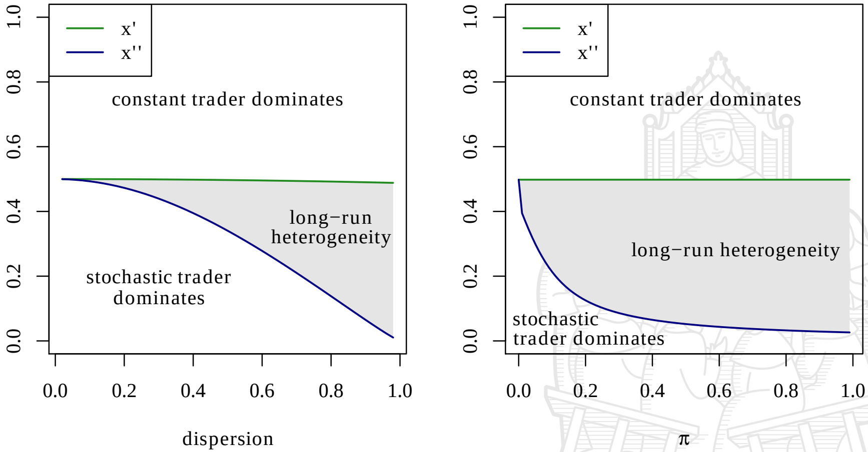

Outline

Concluding remarks

- trade-off between portfolio riskiness and variability
- $\exists$ intrinsic penalty in adopting stochastic strategy
- a constant trader can dominate a stochastic trader with a on-average safer portfolio
- a constant trader can invade a stochastic trader with an always-strictly-safer portfolio
- the stochastic trader (fire-)sells at low price and buys at high price
- generic long-run heterogeneity
- endogenous fluctuations of price and wealth
- volatility clustering
- loss of generality in adopting the *deterministic skeleton* approach

Thank you very much!

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…and props to the European Project PIOF-GA-2011-300637 MSAEO for financial support