Wealth-driven asymptotic survival in a financial market with demand shocks

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This version: 14th June 2017
1 Context and motivation

2 The model

3 Long-run outcomes

4 Simulation

5 Concluding remarks
Friedman ‘as if’ argument (1953)
- economic agents can be described as if they were fully rational
- non-rational agents would get wiped out of the market

Empirical evidence
- trade occurs
- excess volatility
- market sentiment

Heterogeneous Agent Models (HAMs)
- analytical investigation of long-run dynamics
- incorporate various degrees of bounded rationality
traditional HAMs

- feedback mechanisms from realised market outcomes
  - e.g. fundamentalist vs. chartist
- long-run analysis within a notion of equilibrium
  - i.e. actions are fixed at the equilibrium
- ‘deterministic skeleton’ approach

what happens if one introduces *persistent* demand shocks?

**Black (1986) JoF**

*noise traders:* “trading on noise as if it were information”

**Noise traders vs. rational arbitrageurs**

- De Long et al. (1990) JPE price impact, fixed wealth
- De Long et al. (1991) JoB endogenous wealth, no price impact
Context and motivation

Our contribution

- constant vs. stochastic portfolio strategies
- wealth-driven market selection
- conditions for survival and dominance

Main findings

- trade-off between portfolio riskiness and variability
- long-run heterogeneity
- non-trivial price-wealth dynamics (e.g. volatility clustering)
- loss of generality in adopting the deterministic skeleton approach
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The model

Consider a stylised financial market (no consumption)

- trade takes place in discrete time $t$
- risk-free bond
  - perfectly elastic supply
  - constant $r_f$ return
  - price normalised to 1 (numéraire)
- long-lived risky security
  - positive dividend $d_t = d_{t-1}(1 + g)$, $g > r_f$
  - unitary constant supply
  - market clearing price $p_t$

At each time step $t$ trader $n$

- invests a fraction $x_{n,t}$ of her wealth $w_{n,t}$ into the risky security
- residually invests $(1 - x_{n,t}) \cdot w_{n,t}$ into the bond

Trader $n$ wealth $w_{n,t}$ equals the current market value of her portfolio
Traders’ behaviour

Assumption

- **constant trader** always invests $\bar{x} \in (0, 1)$
- **stochastic trader** invests according to Markov process $\{X_t, \ t \in \mathbb{N}\}$

\[\begin{align*}
1 - \pi^d & \quad \xrightarrow{\pi^d} \quad \x^u \\
\pi^u & \quad \x^d \\
\pi^u & \quad \x^d \\
\end{align*}\]

- $\pi^u > 0$
- $\pi^d > 0$
- $0 < x^d < x^u < 1$
Assumption

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X^u & \quad \pi^u \\
X^d & \quad 1 - \pi^u
\end{align*}
\]

\[
\pi^u > 0 \\
\pi^d > 0 \\
0 < X^d < X^u < 1
\]
Laws of motion of the economy

\[ \mathcal{F}_{x_{t-1}, x_t} : \mathcal{D} \to \mathcal{D}, \quad \mathcal{D} = \Delta \times (-1, +\infty) \times \mathbb{R}_{++} \]

\[
\begin{align*}
\varphi_t &= \varphi_{t-1} \frac{1 + x_{t-1} (r_t + e_t)}{1 + (r_t + e_t) [\varphi_{t-1} x_{t-1} + (1 - \varphi_{t-1}) \bar{x}]} \\
r_t &= \frac{\varphi_{t-1} [x_t (1 + e_t x_{t-1}) - x_{t-1}] + (1 - \varphi_{t-1}) e_t \bar{x}^2}{\varphi_{t-1} x_{t-1} (1 - x_t) + (1 - \varphi_{t-1}) \bar{x} (1 - \bar{x})} \\
e_t &= e_{t-1} \frac{1 + g}{1 + r_{t-1}}
\end{align*}
\]

where

\[ \varphi_t = \frac{\text{wealth of stochastic trader}}{\text{total wealth}} \in [0, 1] \]
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4. Simulation
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Long-run outcomes

Terminology

- trader \( n \) is said to \textit{survive} on \( \{x_t\}_{t=0}^\infty \) if \( \limsup_{t \to \infty} \varphi_{n,t} > 0 \)

- trader \( n \) is said to \textit{vanish} on \( \{x_t\}_{t=0}^\infty \) if \( \limsup_{t \to \infty} \varphi_{n,t} = 0 \)

- trader \( n \) is said to \textit{dominate} on \( \{x_t\}_{t=0}^\infty \) if \( \liminf_{t \to \infty} \varphi_{n,t} = 1 \)

Proposition

If \( (\varphi^*, r^*, e^*) \in \mathcal{D} \) is a fixed point of system \( \mathcal{F} \) then either

1. the constant trader dominates (steady state \( \mathcal{C} \))
2. the stochastic trader dominates (steady state \( \mathcal{S} \))
Terminology

- Trader $n$ is said to survive on $\{x_t\}_{t=0}^{\infty}$ if $\limsup_{t \to \infty} \varphi_{n,t} > 0$
- Trader $n$ is said to vanish on $\{x_t\}_{t=0}^{\infty}$ if $\limsup_{t \to \infty} \varphi_{n,t} = 0$
- Trader $n$ is said to dominate on $\{x_t\}_{t=0}^{\infty}$ if $\liminf_{t \to \infty} \varphi_{n,t} = 1$

Proposition

If $(\varphi^*, r^*, e^*) \in \mathcal{D}$ is a fixed point of system $\mathcal{F}$ then either

1. the constant trader dominates (steady state $\mathcal{C}$)
2. the stochastic trader dominates (steady state $\mathcal{S}$)
A dominant constant trader

\[ \varphi^c = 0 \quad r^c = g \quad e^c = g \frac{1 - \bar{x}}{\bar{x}} \]

Proposition

Steady state \( C \) is locally asymptotically stable if

\[ \lambda^c = \frac{(\bar{x} + g\pi^u)^{\frac{\pi^u}{\pi^u + \pi^d}}(\bar{x} + g\pi^d)^{\frac{\pi^d}{\pi^u + \pi^d}}}{\bar{x}(1 + g)} < 1 \]
A dominant constant trader

Sufficient conditions

- $\bar{x} \geq x^u \implies \mathcal{C}$ is locally asymptotically stable
- $\bar{x} \leq x^d \implies \mathcal{C}$ is unstable
A dominant constant trader

![Graphical representation of the stability of a dominant constant trader](chart)

**Proposition**

\[ \exists! \ x' \in (0, 1) \text{ such that } \]

- \( \forall \ x < x', C \) is unstable
- \( \forall \ x > x', C \) is locally asymptotically stable

Moreover, \( x^d < x' < x^u \)
A dominant constant trader

\[ C \text{ is unstable} \quad \text{and} \quad C \text{ is loc. as. stable} \]

\[ 0 \quad x^d \quad x' \quad x^u \quad 1 \]

Special case

\[ \pi^u = \pi^d \quad \implies \quad x' = \frac{x^u + x^d}{2} - h(g) \]

\[ h(0) = 0 \]

\[ h(g) > 0 \]

\[ h'(g) > 0 \]

Proposition

\[ x' < \mathbb{E}[x] \quad \text{even if} \quad \pi^u \neq \pi^d \]
A dominant constant trader

\[ \mathcal{C} \text{ is unstable} \quad \text{loc. as. stable} \]

\[ 0 \quad x^d \quad x' \quad \mathbb{E}[\mathcal{X}] \quad x^u \quad 1 \]

**Special case**

\[ \pi^u = \pi^d \implies x' = \frac{x^u + x^d}{2} - h(g) \]

\[ h(0) = 0 \quad h(g) > 0 \quad h'(g) > 0 \]

**Proposition**

\[ x' < \mathbb{E}[\mathcal{X}] \quad \text{even if} \quad \pi^u \neq \pi^d \]
A dominant stochastic trader

It holds $r^{ud} < g < r^{du}$ and $e^{u} < e^{d}$. Moreover, ‘usually’ $r^{ud} < 0$.
A dominant stochastic trader

**Proposition**

Steady state $S$ is locally asymptotically stable if

$$
\lambda^S = \frac{1}{1 + g} \left[ 1 + \frac{g \bar{x}}{x^u} \right] \frac{\pi^u (1 - \pi^d)}{\pi^u + \pi^d}
$$

$$
\cdot \left[ \frac{\bar{x} \left[ g (1 - x^u) - (x^u - x^d) \right] + x^u (1 - x^d)}{x^u (1 - x^u)} \right] \cdot \frac{\pi^u \pi^d}{\pi^u + \pi^d}
$$

$$
\cdot \left[ 1 + \frac{g \bar{x}}{x^d} \right] \frac{\pi^d (1 - \pi^u)}{\pi^u + \pi^d} < 1
$$
A dominant stochastic trader

Numerical result

∃! \( x'' \in (0, 1) \) such that

- \( \forall \bar{x} < x'', \mathcal{S} \) is locally asymptotically stable
- \( \forall \bar{x} > x'', \mathcal{S} \) is unstable
Proposition

\[ \exists \, \hat{g} > 0 \text{ such that } \forall \, g < \hat{g} \text{ it holds } x'' < x^d < x'. \] In particular

\[ \hat{g} = \frac{x^u - x^d}{1 - x^u} \]
Long-run heterogeneity

$S$ is loc. as. stable
stochastic investor
dominates

$C$ and $S$ are unstable
long-run
eheterogeneity

$C$ is loc. as. stable
constant investor
dominates

$x'' < x'$ holds

- $\forall x^u, x^d, \pi^u, \pi^d \in \{0.01, 0.02, \ldots, 0.99\}$ such that $x^d < x^u$
- $\forall g = \hat{g} \cdot 10^k, k \in \mathbb{N}_+ \text{ such that } g \leq 10$
Corollary

1. there exists a non-degenerate interval \((x'', x')\) such that both \(C\) and \(S\) are unstable \(\forall \bar{x} \in (x'', x')\)

2. the constant trader is able to dominate the stochastic trader even adopting a on-average safer position \(\bar{x} < \mathbb{E}[X]\)

3. the constant trader is able to invade the stochastic trader even adopting an always strictly safer position \(\bar{x} < x^d < x^u\) when \(x'' < x^d\)
Corollary

1. There exists a non-degenerate interval \((x'', x')\) such that both \(C\) and \(S\) are unstable \(\forall \overline{x} \in (x'', x')\).

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\(\text{Asymptotic survival in a financial market with demand shocks} \)

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14th June 2017
Corollary

1. there exists a non-degenerate interval \((x'', x')\) such that both \(\mathcal{C}\) and \(\mathcal{S}\) are unstable \(\forall \; \bar{x} \in (x'', x')\)

2. the constant trader is able to dominate the stochastic trader even adopting a on-average safer position \(\bar{x} < \mathbb{E}[\mathcal{X}]\)

3. the constant trader is able to invade the stochastic trader even adopting an always strictly safer position \(\bar{x} < x^d < x^u\) when \(x'' < x^d\)
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## Initialisation

<table>
<thead>
<tr>
<th>Description</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>stochastic investment up</td>
<td>$x^u$</td>
<td>0.7</td>
</tr>
<tr>
<td>stochastic investment down</td>
<td>$x^d$</td>
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<tr>
<td>probability down when up</td>
<td>$\pi^d$</td>
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<tr>
<td>probability up when down</td>
<td>$\pi^u$</td>
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<tr>
<td>constant investment</td>
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</tr>
<tr>
<td>dividend rate of growth</td>
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</tr>
<tr>
<td>initial wealth share</td>
<td>$\varphi_0$</td>
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</tr>
<tr>
<td>initial return</td>
<td>$r_0$</td>
<td>0.0</td>
</tr>
<tr>
<td>initial yield</td>
<td>$e_0$</td>
<td>0.01</td>
</tr>
</tbody>
</table>
Simulation

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A tale of two regimes

1st regime: \( g = 0.05 \) and \( \overline{x} = 0.45 \)

2nd regime: \( g = 0.005 \) and \( \overline{x} = 0.25 \)
Sensitivity analysis

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Sensitivity analysis

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Concluding remarks

- trade-off between portfolio riskiness and variability
- ∃ intrinsic penalty in adopting stochastic strategy
- a constant trader can dominate a stochastic trader with a on-average safer portfolio
- a constant trader can invade a stochastic trader with an always-strictly-safer portfolio
- the stochastic trader (fire-)sells at low price and buys at high price
- generic long-run heterogeneity
- endogenous fluctuations of price and wealth
- volatility clustering
- loss of generality in adopting the deterministic skeleton approach
Hope you slept comfortably

Thank you very much!

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…and props to the European Project PIOF-GA-2011-300637 MSAEO for financial support