

# An agent-based model of intra-day financial markets dynamics

**Jacopo Staccioli**<sup>†</sup>

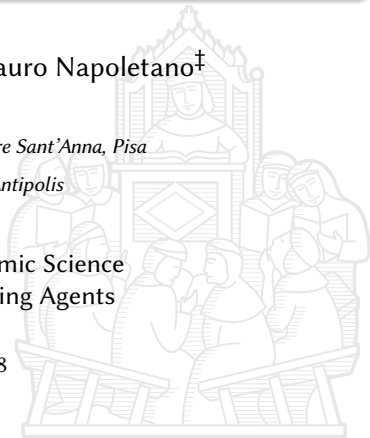
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with Heterogeneous Interacting Agents

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- 1 **Context and motivation**
- 2 Stylised facts
- 3 The model
- 4 Simulations
- 5 Concluding remarks



during recent years, the availability of advanced technology has been substantially reducing the latency required to operate on financial markets, fostering market activity at increasingly higher frequencies

Cont (2011) time to execution dropped 25-fold between 2000 and 2010

Carrion (2013) 68.3% of NASDAQ dollar turnover attributable to HFT

Aldridge (2013) the majority of HFTs delivered positive returns in 2008, whereas 70% of LFTs lost money

## high-frequency traders

- high # of trades per day
- low average gain per trade
- low overnight inventories

## pros? cons?

- market quality
- volatility
- flash-crash

# Stylised facts

## low- or cross-frequency

- properties of returns
- properties of volumes

## high-frequency

- properties of timing and order-flow

## agent-based models

- no model has yet addressed the high-frequency set of stylised facts
- difficulty in mapping simulation time into calendar time

some of the stylised facts have been already (singularly) investigated and linked to patterns of information diffusion

## our proposal

- parsimonious financial agent-based model
- intra-day financial dynamics
- no role for information diffusion
- most of the stylised facts *jointly* emerge from the endogenous interaction of heterogeneous traders

1 Context and motivation

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**SF1 LEPTOKURTOSIS** unconditional distribution displays heavier tail w.r.t. Gaussian distribution

**SF2 NO LINEAR AUTOCORRELATION** positive autocorr. quickly fading away

**SF3 VOLATILITY CLUSTERING** positive autocorr. of absolute/squared value slowly fading away

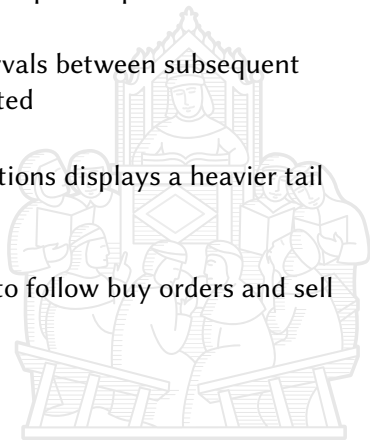
**SF4 LEVERAGE EFFECT** volatility is higher during price drops than during price surges  $\implies$  negative correlation between volatility (absolute returns) and returns

SF5 # PRICE CHANGES PER DAY 10,000+ for blue-chips in liquid markets

SF6 AUTOCORRELATION OF DURATIONS time intervals between subsequent trades are positively autocorrelated

SF7 FAT-TAILED DURATIONS distribution of durations displays a heavier tail w.r.t. exponential distribution

SF8 ORDER-FLOW CLUSTERING buy orders tend to follow buy orders and sell orders tend to follow sell orders

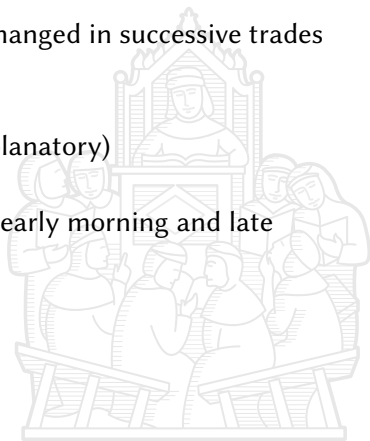




**SF9 VOLUMES AUTOCORRELATION** quantities exchanged in successive trades exhibit positive autocorrelation

**SF10 VOLUME/VOLATILITY CORRELATION** (self-explanatory)

**SF11 U-SHAPED ACTIVITY** volumes peak during early morning and late afternoon



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# Ingredients

- order-driven financial market
    - price-time priority
  - single long-lived security
  - no dividend
  - no fundamental news
  - $N$  heterogeneous agents
    - **fundamentalists**    mean-reverters
    - **chartists**        trend-followers and contrarians
  - no strategy switching
- strict global schedule  $\implies$  Euronext
- endogenous participation based on past volatility
- automatic order cancellation



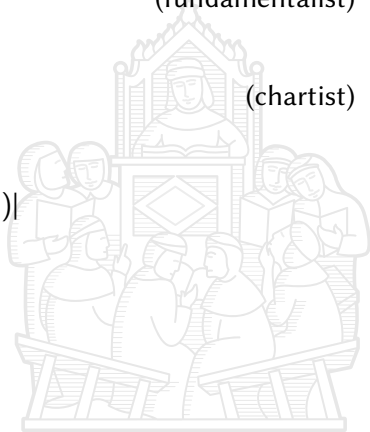
$$\hat{r}_{i,t+h}^F = w_i^F \cdot \log\left(\frac{p^F}{p_t}\right) + \varepsilon_t$$

(fundamentalist)

$$\hat{r}_{i,t+h}^C = w_i^C \cdot \log\left(\frac{p_t}{p_{t-h}}\right) + \varepsilon_t$$

(chartist)

- fundamentalist sensitivity  $w_i^F \sim |\mathcal{N}(0, \sigma_F^2)|$
- chartist sensitivity  $w_i^C \sim \mathcal{N}(\mu_C, \sigma_C^2)$
- fundamental price  $p^F > 0$
- memory/horizon  $h \in \mathbb{N}_+$
- common *i.i.d.* noise  $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$



## definition

a limit order submitted by trader  $i$  at time  $t$  is a triple

$$\begin{aligned}\ell_{i,t} &= \{ \text{price, quantity, validity} \} \\ &= \left\{ \text{round}(p_t \cdot \exp(\hat{r}_{i,t+h}), \text{tick}), \text{sgn}(\hat{r}_{i,t+h}), t+h \right\}\end{aligned}$$

- $\text{round}(\cdot)$  is the rounding function
- $\text{tick}$  is the minimum price increment/decrement
- $\text{sgn}(\cdot)$  is the sign function

- no feedback from current time of the day

## automatic cancellation

a stored order  $\ell_{i,t}$  is automatically deleted from the book

- at its expiration time  $t + h$
- if  $i$  submits a new order with different sign (side)
- if  $i$  submits a new order and  $\ell_{i,t}$  is deemed unfavourable
  - new buy order at lower price
  - new sell order at higher price



# Trader participation

## uniform activation

- exactly one trader is activated at each time step, randomly selected from the population  $N$

## endogenous activation

- trader  $i$  is active at time  $t$  if

$$|r_\tau| > \delta_{i,t} \sim |\mathcal{N}(0, \sigma_\delta^2)|$$

where  $\tau < t$  denotes the last time a trade occurred

- if  $|r_\tau| < \delta_{i,t}, \forall i = 1, \dots, N$  then uniform activation with probability  $\phi > 0$

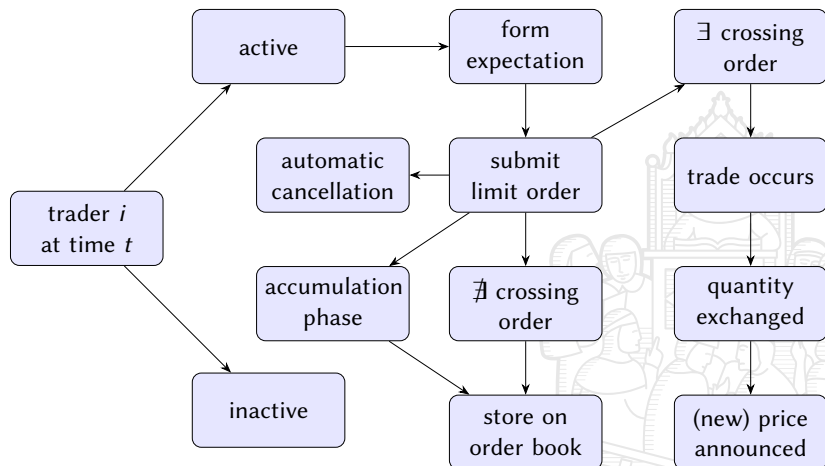
# Timing (EURONEXT)

<b>time</b>	<b>phase</b>	<b>duration</b>
from 7:15am to 9:00am	pre-opening	6,300 s
at 9:00am	opening auction	—
from 9:00am to 5:30pm	main trading session	30,600 s
from 5:30pm to 5:35pm	pre-closing	300 s
at 5:35pm	closing auction	—
10 hours, 20 minutes	—	37,200 s

**1 simulation step**  $\iff$  **1 calendar second**



# Workflow



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# Preliminaries

we simulate the model numerically under three scenarios

**NT** : only noise traders

**FC** : fundamentalists and chartists with uniform participation

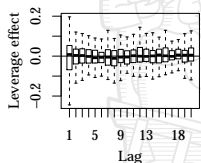
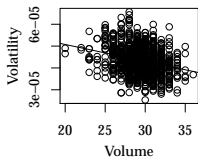
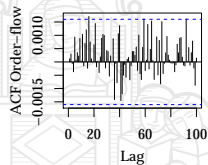
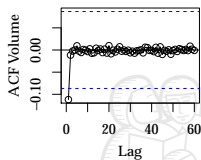
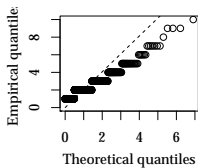
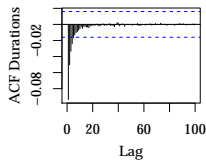
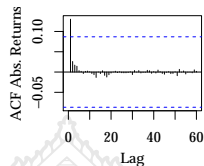
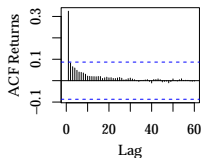
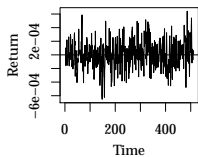
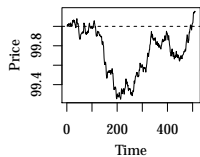
**EA** : fundamentalists and chartists with endogenous participation

parameter	value
$N$	1,000
$p^F$	100
tick	0.001
$h$	1,000
$p_0$	$p^F$

at the beginning of the simulation all chartists are provided a history of past prices between  $t = -h$  and  $t = 0$  that evolves (backwards) as a pure random walk

- irregular time series are pooled into minute-by-minute data
- results are averaged across 100 Montecarlo simulations

# NT scenario



▶ param

avg. # of trades = 14,958

$\kappa \approx 3.17$

▶ zoom



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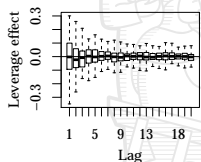
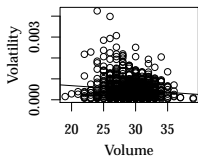
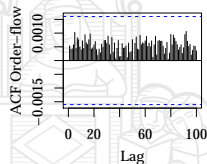
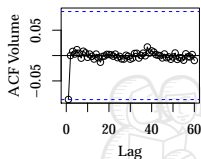
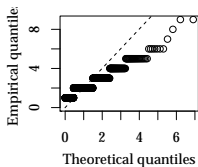
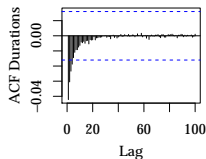
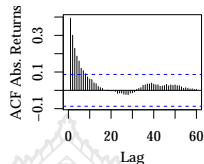
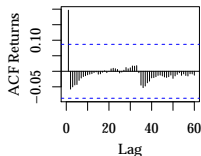
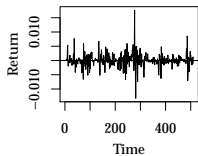
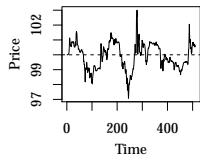
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# FC scenario



▶ param

avg. # of trades = 14,953

$\kappa \approx 14.5$

▶ zoom



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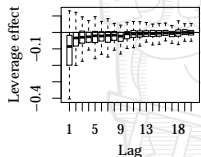
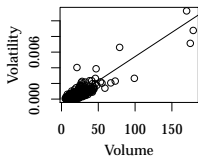
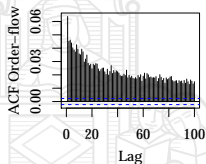
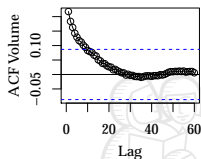
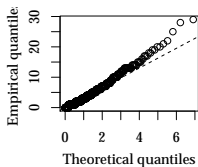
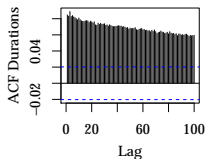
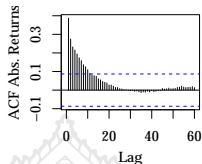
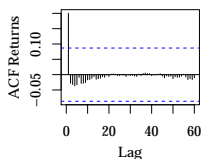
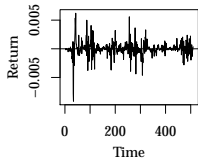
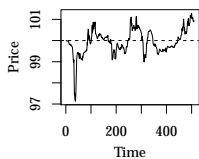
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# EA scenario



▶ param

avg. # of trades = 9,991

$\kappa \approx 13.96$

▶ zoom



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# Stylised facts

stylised fact	scenario			
	NT	FC	EA	
SF1	leptokurtic returns	X	✓	✓
SF2	no linear autocorr.	✓	✓	✓
SF3	volatility clustering	X	✓	✓
SF4	leverage effect	X	X	X
SF5	# price changes	✓	✓	✓
SF6	autocorr. durations	X	X	✓
SF7	fat-tailed durations	X	X	✓
SF8	order-flow clustering	X	X	✓
SF9	autocorr. volumes	X	X	✓
SF10	volume/volatility corr.	X	X	✓
SF11	U-shaped activity	X	X	X



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## assumptions

- no information diffusion – everything is common knowledge
- trading emerges as the consequence of differing (stable) beliefs
  - fundamentalists vs. chartists
- strict timing and microstructure from EURONEXT
- endogenous participation based on past volatility

## results

- NT slight dependence in returns quickly fading, # trades/day
- FC [NT]  $\oplus$  leptokurtosis and volatility clustering
- EA [FC]  $\oplus$  dependence in timing, volumes, and order-flow



## conceivable extensions

- time feedback in trading strategies
  - budget constraint/leverage requirement
  - more complex chartist specification  $\Rightarrow$  leverage effect
  - calibration of model parameters  $\Rightarrow$  policy experiments
- }  $\Rightarrow$  U-shaped seasonality

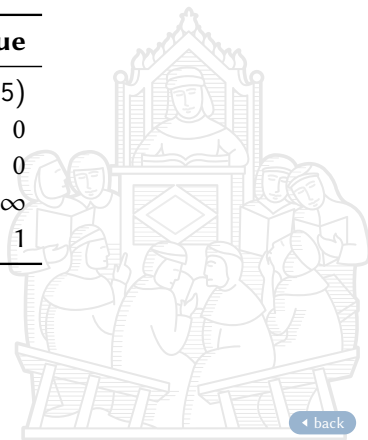
# Thank you very much!

[j.staccioli<at>santannapisa.it](mailto:j.staccioli@santannapisa.it)

## Dolphins project

...and props to the European Project 640772 - DOLFINS - H2020-FETPROACT-2014  
for financial support

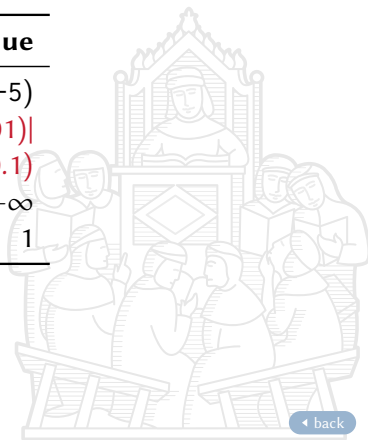
param.	value
$\varepsilon_t$	$\mathcal{N}(0, 5e-5)$
$w_i^F$	0
$w_i^C$	0
$\delta_t$	$+\infty$
$\phi$	1



◀ back



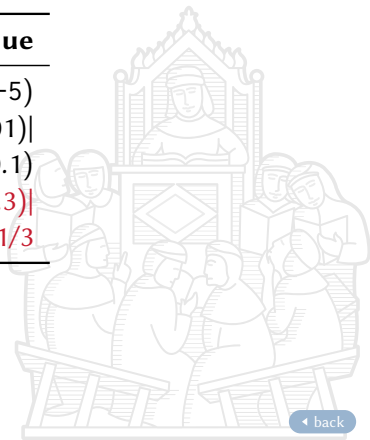
param.	value
$\varepsilon_t$	$\mathcal{N}(0, 5e-5)$
$w_i^F$	$ \mathcal{N}(0, 0.001) $
$w_i^C$	$\mathcal{N}(0.01, 0.1)$
$\delta_t$	$+\infty$
$\phi$	1



◀ back



param.	value
$\varepsilon_t$	$\mathcal{N}(0, 5e-5)$
$w_i^F$	$ \mathcal{N}(0, 0.001) $
$w_i^C$	$\mathcal{N}(0.01, 0.1)$
$\delta_t$	$ \mathcal{N}(0, 0.3) $
$\phi$	$1/3$



◀ back



# NT scenario (i)

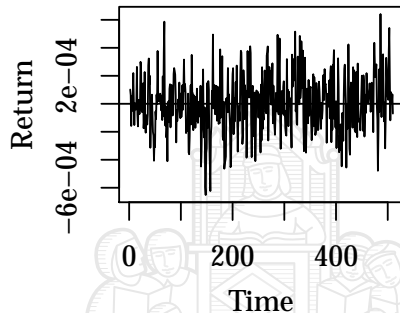
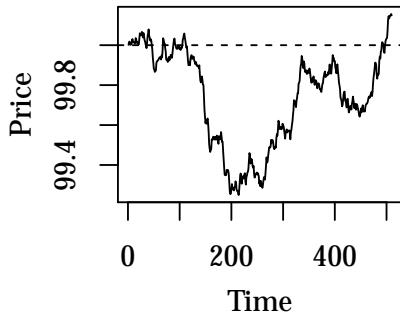


Figure: Price series (left) and return series (right) for a typical trading day

## NT scenario (ii)

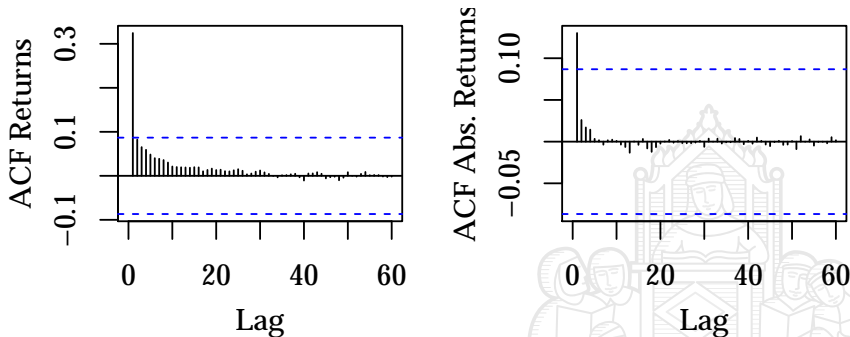


Figure: Autocorrelation of returns (left) and of absolute returns (right)





# NT scenario (iii)

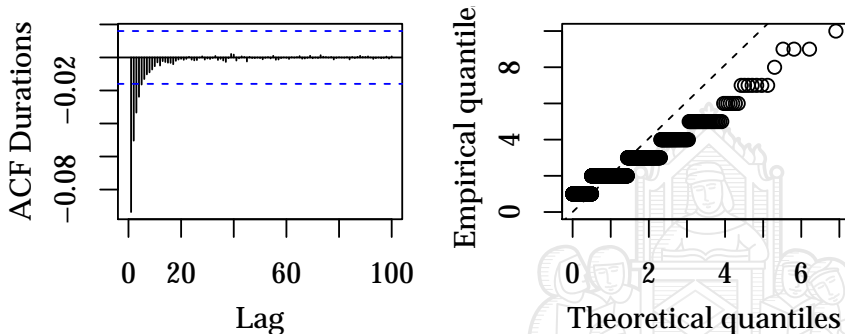


Figure: Autocorrelation of durations (left) and Q-Q of their distribution (right)

# NT scenario (iv)

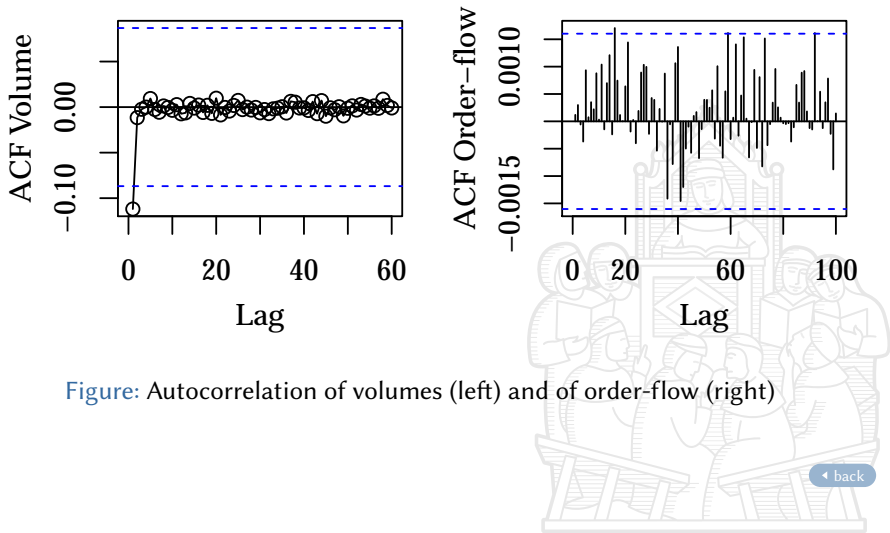


Figure: Autocorrelation of volumes (left) and of order-flow (right)

# NT scenario (v)

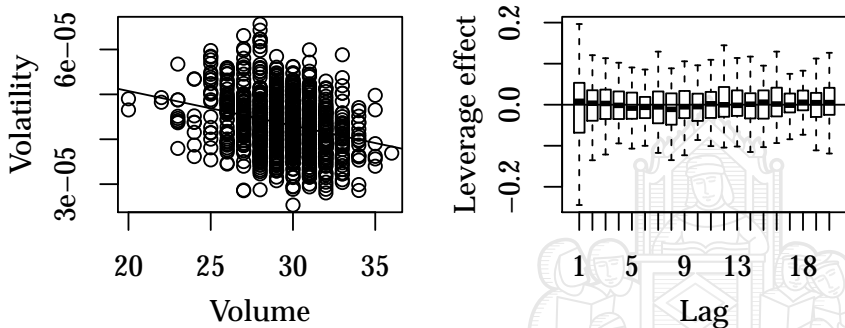


Figure: Volume/volatility correlation (left) and leverage effect (right)



# FC scenario (i)

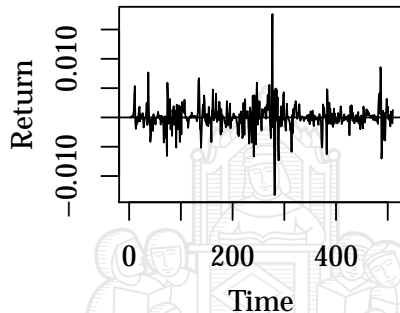
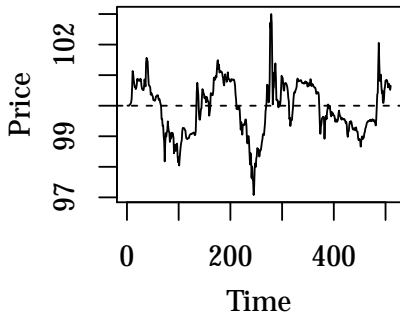


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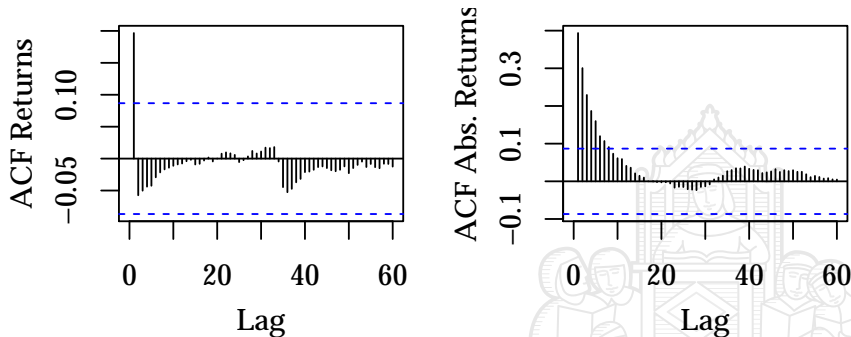


Figure: Autocorrelation of returns (left) and of absolute returns (right)

## FC scenario (iii)

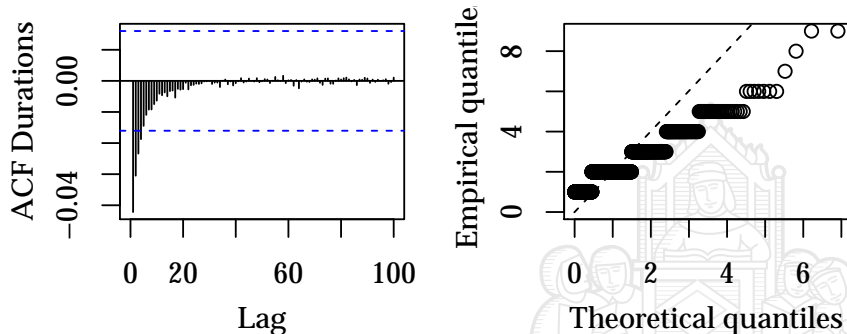


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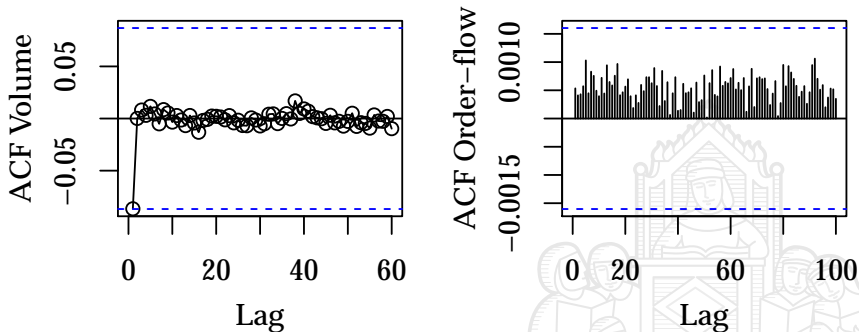


Figure: Autocorrelation of volumes (left) and of order-flow (right)

◀ back



# FC scenario (v)

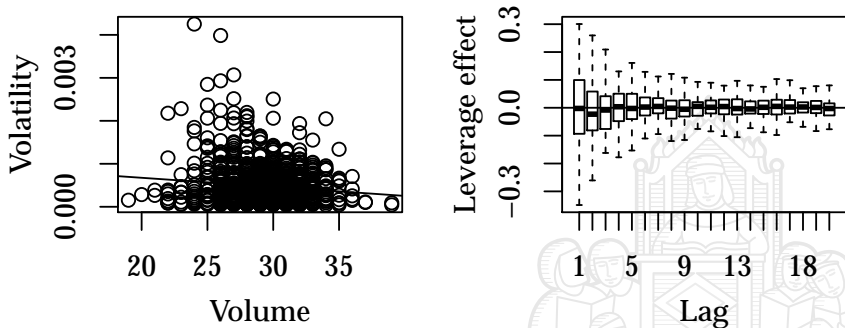


Figure: Volume/volatility correlation (left) and leverage effect (right)



# EA scenario (i)

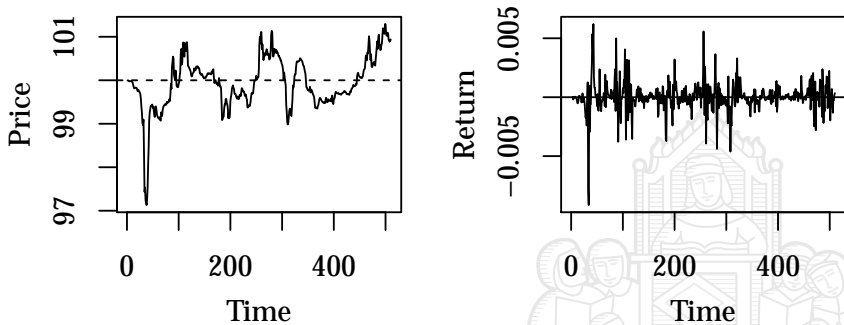


Figure: Price series (left) and return series (right) for a typical trading day

## EA scenario (ii)

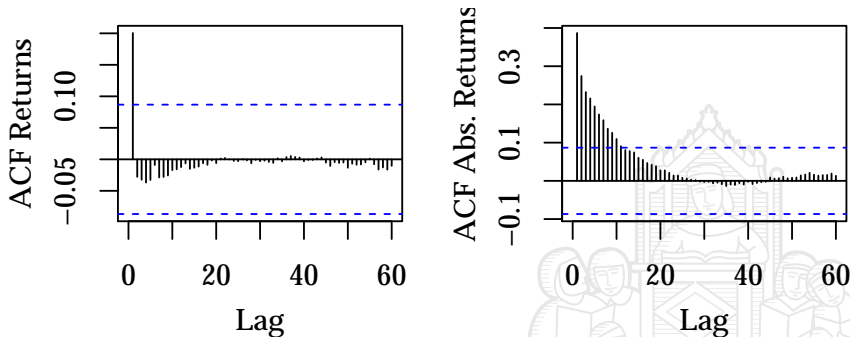


Figure: Autocorrelation of returns (left) and of absolute returns (right)

## EA scenario (iii)

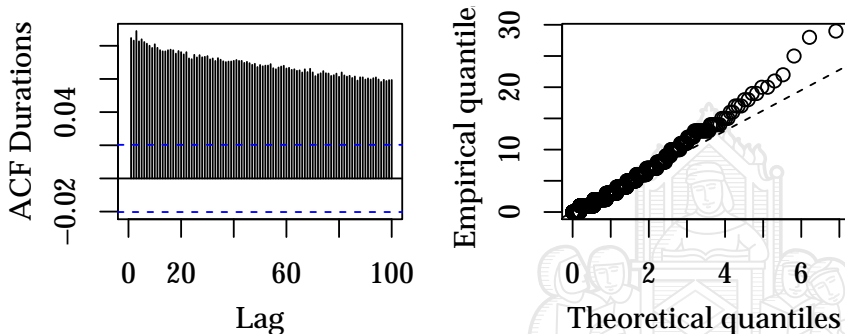


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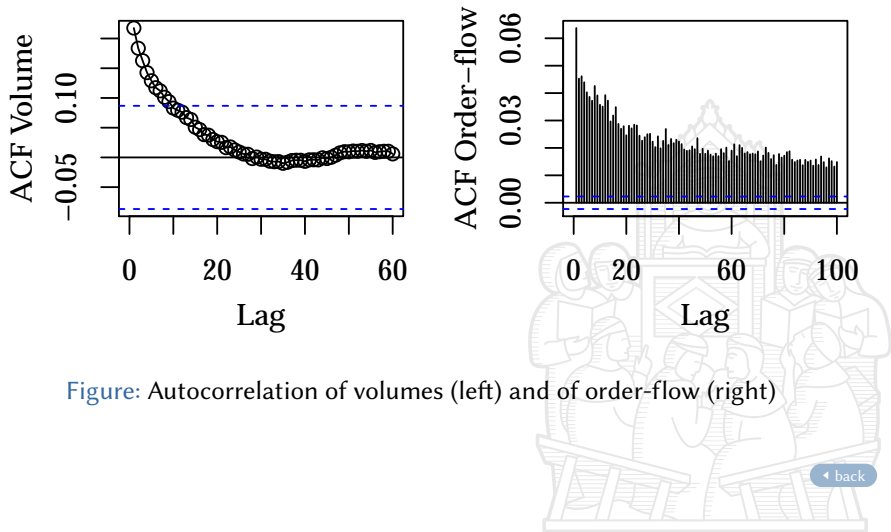


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# EA scenario (v)

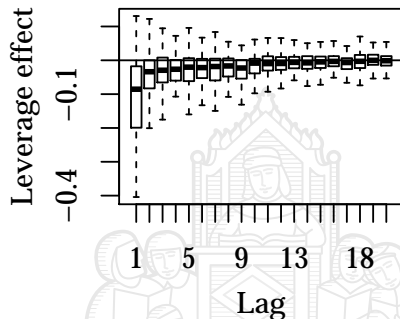
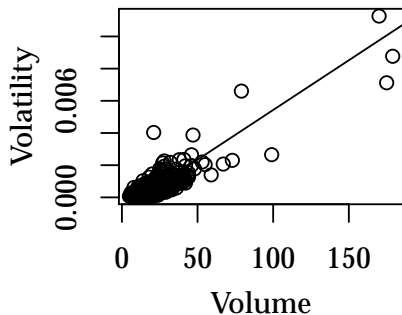


Figure: Volume/volatility correlation (left) and leverage effect (right)