

Bubble-and-bust dynamics under walrasian asset pricing and heterogeneous traders

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Financial bubbles



FRED 

— NASDAQ Composite Index©



Source: NASDAQ OMX Group

Shaded areas indicate US recessions - 2014 research.stlouisfed.org

“if the reason that the price is high today is only because investors believe that the selling price will be high tomorrow – when ‘fundamental’ factors do not seem to justify such a price – then a bubble exists” [Stiglitz, 1990]

“a sharp rise in the price of an asset or a range of assets in a continuous process, with the initial rise generating expectations of further rises and attracting new buyers – generally speculators interested in profits from trading in the asset rather than its use or earnings capacity” [Kindleberger, 1978]

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Our proposal

We set up a model able to yield:

- *endogenous* bubble-and-bust dynamics
- as a result of the sole interaction among *heterogeneous adaptive traders*
- highlighting booms and crashes as intrinsic features of financial markets

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- analytical investigations of the dynamical systems representing the laws of motion of the economy
- analytical tractability often leads to simplifying assumptions
- focus on asymptotic properties

Agent-Based Models

- computational (numerical) study of economies modelled as evolving systems of interacting agents
- complex behaviour specifications
- keep track of the whole dynamics

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Consider a pure-exchange economy:

- N heterogeneous traders (index $\mathcal{N} = \{1, \dots, n, \dots, N\}$);
- L long-lived risky securities (index $\mathcal{L} = \{1, \dots, l, \dots, L\}$);
- a riskless bond;
- time is discrete (index $t \in \mathcal{T}$);
- risky securities, present in fixed amount, have ex-dividend price p_t^l and pay random dividend d_t^l at the end of each period;
- the bond, inelastically supplied, has price normalized to 1 (*numéraire*) and yields $r_f > 0$ in every t ;
- trader wealth $W_{n,t}$ equals the market value of the portfolio he holds;
- fundamentals are proxied by the dividend yield $e_t^l = \frac{d_t^l}{p_{t-1}^l}, \forall l \in \mathcal{L}$.

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Trader behaviour

At the beginning of each time step, trader n invests a share $x_{n,t}^\ell$ of his wealth in security ℓ ; the decision is made according to the information set

$$\mathcal{I}_t = \{p_\tau^1, \dots, p_\tau^L; d_\tau^1, \dots, d_\tau^L \mid \tau < t\}$$

that is common knowledge, and to trader-specific investment function

$$f_n : \mathbb{R}^{\tau \times L} \longrightarrow \mathbb{R}^L \quad \text{such that} \quad \mathbf{x}_{n,t} = f_{n,t}(\mathcal{I}_t)$$

that is independent on wealth, coherent with CRRA attitude.

At every time step, each trader faces an optimisation problem of the form:

$$\max_{\mathbf{x}_{n,t}} \mathbb{E} \left[\frac{W_{n,t}^{1-\gamma_n} - 1}{1 - \gamma_n} \right]$$

s.t.

$$W_{n,t} = W_{n,t-1} \cdot \left[x_{n,t-1}^0 \cdot (1 + r_f) + \sum_{\ell=1}^L x_{n,t-1}^\ell \cdot \left(\frac{p_t^\ell}{p_{t-1}^\ell} + e_t^\ell \right) \right]$$

where $\gamma_n > 0$ denotes the risk-aversion coefficient.

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We assume the trader forms expectations about future price returns and their (co)variances by means of EWMA predictors over the information set previously defined:

$$\hat{\rho}_{n,t}^{\ell} = \lambda_n \cdot \sum_{\tau=0}^{\infty} (1 - \lambda_n)^{\tau} \cdot \rho_{t-\tau-1}^{\ell}$$

$$\hat{\sigma}_{\rho,n,t}^{\ell,h} = \lambda_n \cdot \sum_{\tau=0}^{\infty} (1 - \lambda_n)^{\tau} \cdot \left[\rho_{t-\tau-1}^{\ell} - \hat{\rho}_{n,t-\tau-1}^{\ell} \right] \cdot \left[\rho_{t-\tau-1}^h - \hat{\rho}_{n,t-\tau-1}^h \right]$$

where $\rho_t^{\ell} = \frac{p_t^{\ell}}{p_{t-1}^{\ell}} - 1$ is the price return of security ℓ between $t - 1$ and t . The memory decay factor $\lambda_n \in (0, 1)$ captures the way relative weights are distributed across more recent and older observations.

Trader investment function

We adopt the same mean-variance approximation of the optimal investment function proposed in Chiarella and He (2001):

$$\mathbf{x}_{n,t} = \frac{1}{\gamma_n} \cdot \mathbb{C}_{n,t}^{-1} \cdot \left[\mathbb{E}_{n,t} - r_f \cdot \mathbf{1} \right]$$

$$\mathbb{E}_{n,t}^{\ell} = \bar{e}^{\ell} + d_n \cdot \hat{\rho}_{n,t}^{\ell}$$

$$\mathbb{C}_{n,t}^{\ell,h} = \hat{\sigma}_{\rho,n,t}^{\ell,h} + \sigma_e^{\ell,h}$$

where $\mathbb{E}_{n,t}$ and $\mathbb{C}_{n,t}$ are, respectively, the vector of expected total returns and the expected variance-covariance matrix and d_n is a behavioural parameter:

- $d_n = 0$ trader n is a fundamentalist
- $d_n > 0$ trader n is a trend-chaser
- $d_n < 0$ trader n is a trend-contrarian

Assumption

e_t^{ℓ} is drawn at each time step from a L -dimensional known probability distribution with mean \bar{e} and covariance matrix Σ .

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Proposition

If short positions are not allowed, i.e.

$$x_{n,t}^{\ell} \in (0, 1) \quad \forall n \in \mathcal{N}, \forall \ell \in \mathcal{L}, \forall t \in \mathcal{T}$$

then prevailing prices exist, are unique and strictly positive. It holds:

$$p_t^{\ell} = p_{t-1}^{\ell} \cdot \frac{x_t^{\ell}}{x_{t-1}^{\ell}} \cdot \frac{x_{t-1}^0 \cdot (1 + r_f) + \sum_{\ell=1}^L x_{t-1}^{\ell} \cdot e_t^{\ell}}{x_t^0}$$

Assumption

No trader can take short position in any asset, i.e. the image of traders' investment functions is restricted such that

$$f_n : \mathbb{R}^{\tau \times L} \rightarrow \text{Int}(\Delta^L)$$

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Definition

Individual wealth shares:

$$\varphi_{n,t} = \frac{W_{n,t}}{\sum_{n=1}^N W_{n,t}}$$

- A trader n is said to '*survive*' the economy if his long-run wealth-share is significantly different from 0, i.e. if $\lim_{t \rightarrow \infty} \varphi_{n,t} > 0$
- A trader n is said to '*dominate*' the economy if his long-run wealth-share is significantly close to 1, i.e. if $\lim_{t \rightarrow \infty} \varphi_{n,t} = 1$

Following Anufriev et al. (2006), two types of equilibria are possible:

- 1 Single-survivor equilibria (most '*aggressive*' trader)
- 2 Multiple-survivor equilibria (non-generic)

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Description	Value
Initial population size	$N = 200$
Number of risky assets	$L = 1$
Static population	true
Riskless rate of return	$r_f = 0.02$
γ distribution	$\gamma_n \sim \mathcal{U}(1.0, 1000.0)$
λ distribution	$\lambda_n = 0.1, \forall n \in \mathcal{N}$
d distribution	$d_n = 1.0, \forall n \in \mathcal{N}$
Initial wealth endowment	$W_{n,0} = 50.0, \forall n \in \mathcal{N}$
Yield mean	$\bar{e} = 0.04$
Yield variance	$\sigma_e^2 = 1.0e-4$
Yield realisation distribution	$e_t \sim \mathcal{N}(\bar{e}, \sigma_e^2)$
Initial risky asset price level	$p_0 = 0.1$
x_n admissible interval	$x_{n,t} \in [0.01, 0.99], \forall n \in \mathcal{N}, \forall t$

Table: Parameters and initial conditions (1)

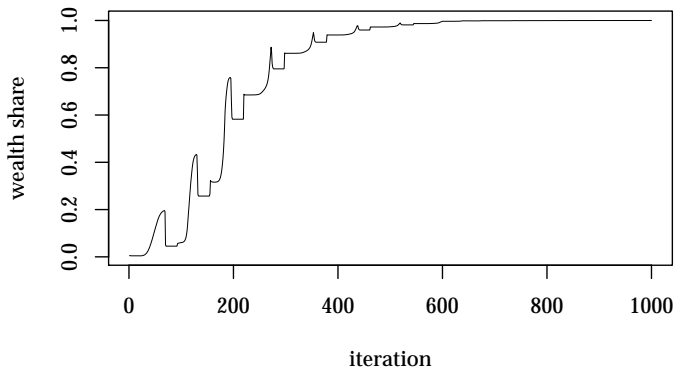


Figure: Evolution of wealth-share for the least-risk-averse trader. Single-survivor.

Description	Value
Initial population size	$N = 200$
Number of risky assets	$L = 1$
Static population	true
Riskless rate of return	$r_f = 0.02$
γ distribution	$\gamma_n \sim \mathcal{U}(100.0, 1000.0)$
λ distribution	$\lambda_n = 0.1, \forall n \in \mathcal{N}$
d distribution	$d_n = 1.0, \forall n \in \mathcal{N}$
Initial wealth endowment	$W_{n,0} = 50.0, \forall n \in \mathcal{N}$
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Table: Parameters and initial conditions (2)

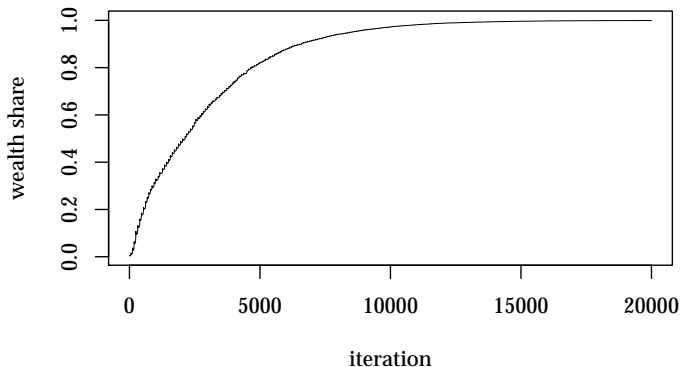
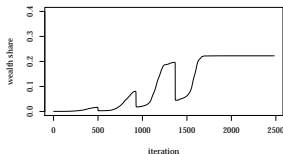


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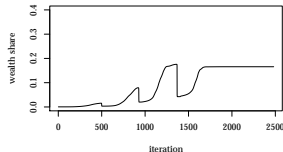
Description	Value
Initial population size	$N = 200$
Number of risky assets	$L = 1$
Static population	true
Riskless rate of return	$r_f = 0.02$
γ distribution	$\gamma_n \sim \mathcal{U}(1.0, 1000.0)$
λ distribution	$\lambda_n = 0.01, \forall n \in \mathcal{N}$
d distribution	$d_n = 1.0, \forall n \in \mathcal{N}$
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Table: Parameters and initial conditions (3)

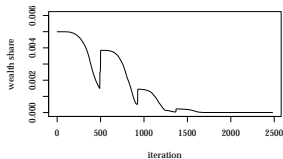
Simulation results - survival patterns (cont'd)



(a) lowest γ_n



(b) second-lowest γ_n



(c) highest γ_n

Figure: Multiple-survivor equilibrium. Evolution of wealth-shares.

Heterogeneity in the risk-aversion coefficient, within the stability domain of the system, triggers a wealth-driven selection mechanism.

- For a short memory-span (large λ), the least risk-averse trader survives and dominates the economy;
- For a long memory-span (small λ), multiple traders, still low-risk averse, survive and display identical investment decisions.

Out of the stability domain of the system (i.e. for large enough λ , following Anufriev et al. 2006) selection does not occur: individual wealth-shares keep fluctuating indefinitely.

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Description	Value
Initial population size	$N = 1000$
Number of risky assets	$L = 1$
Static population	true
Riskless rate of return	$r_f = 0.02$
γ distribution	$\gamma_n \sim \mathcal{U}(1.0, 500.0)$
λ distribution	$\lambda_n = 0.0036, \forall n \in \mathcal{N}$
d distribution	$d_n = 1.0, \forall n \in \mathcal{N}$
Initial wealth endowment	$W_{n,0} = 50.0, \forall n \in \mathcal{N}$
Yield mean	$\bar{e} = 0.04$
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Table: Parameters and initial conditions (4)

Simulation results: price dynamics (cont'd)

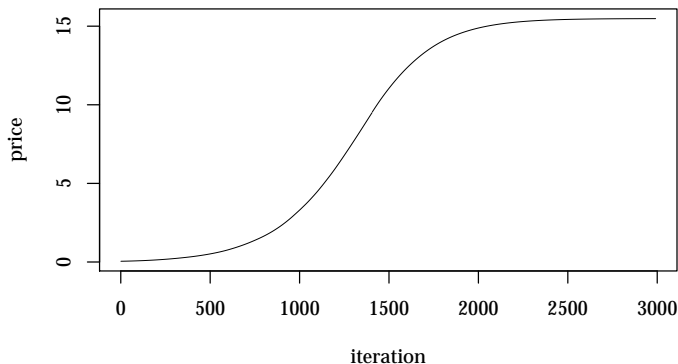


Figure: Price dynamics. Smooth and monotone convergence to equilibrium.

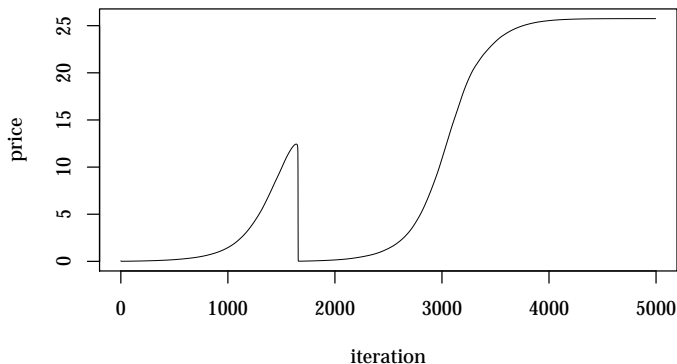


Figure: Price dynamics. Emergence of a bubble-and-bust cycle. $\lambda_n = 0.00365$

Simulation results: price dynamics (cont'd)

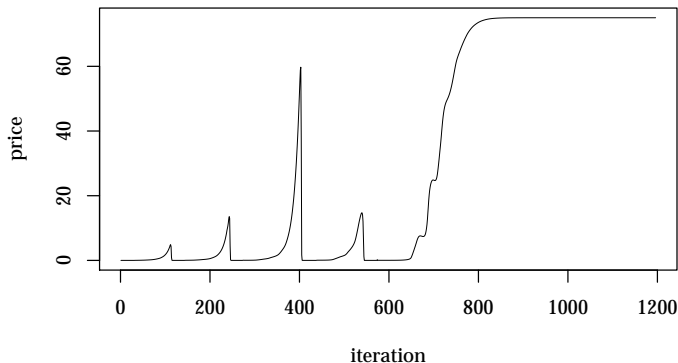


Figure: Price dynamics. Emergence of multiple bubble-and-bust cycles. $\lambda_n = 0.155$

Simulation results: price dynamics (cont'd)

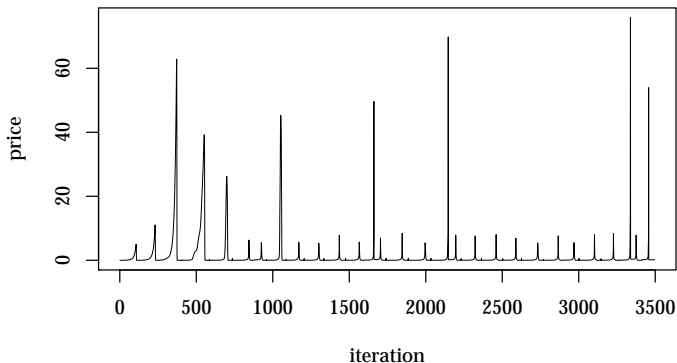


Figure: Price dynamics. No convergence to an equilibrium value. $\lambda_n = 0.16$

The emergent properties observed in a trend-chasers-only setting maintain robustness with respect to the introduction of fundamentalist and trend-contrarian traders in the economy.

- Fundamentalists are expected to stabilise the price as they act against chartists whenever current price deviates from its fundamental value;
- Contrarians shall counteract the attempt made by trend-chasers to exacerbate the price trend by acting in a symmetrical fashion.

We differentiate the d parameter in order to model a population largely composed of quasi-fundamentalists and well balanced crowds of trend-chasers and contrarians.

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Simulation results: price dynamics (cont'd)



We now shift the analysis to the transitional price dynamics:

Description	Value
Initial population size	$N = 1000$
Number of risky assets	$L = 1$
Static population	true
Riskless rate of return	$r_f = 0.02$
γ distribution	$\gamma_n \sim \mathcal{U}(1.0, 500.0)$
λ distribution	$\lambda_n = 0.1, \forall n \in \mathcal{N}$
d distribution	$d_n \sim \mathcal{N}(0, 1)$
Initial wealth endowment	$W_{n,0} = 50.0, \forall n \in \mathcal{N}$
Yield mean	$\bar{e} = 0.04$
Yield variance	$\sigma_e^2 = 1.0e-4$
Yield realisation distribution	$e_t \sim \mathcal{N}(\bar{e}, \sigma_e^2)$
Initial risky asset price level	$p_0 = 0.1$
x_n admissible interval	$x_{n,t} \in [0.01, 0.99], \forall n \in \mathcal{N}, \forall t$

Table: Parameters and initial conditions (5)

Simulation results: price dynamics (cont'd)

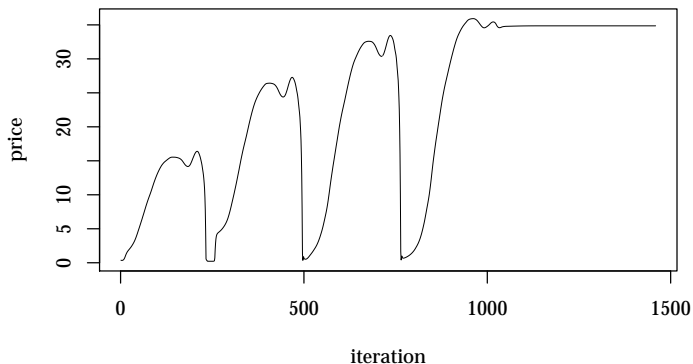


Figure: Price dynamics. Fundamentalists vs. chartists.

Simulation results: price dynamics (cont'd)

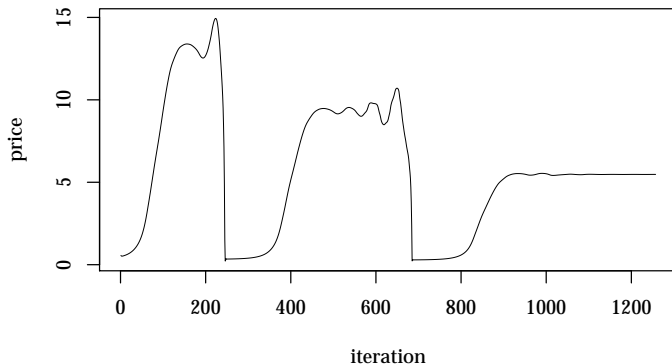


Figure: Price dynamics. Fundamentalists vs. chartists. $\gamma_n \sim \mathcal{U}(1.0, 1000.0)$

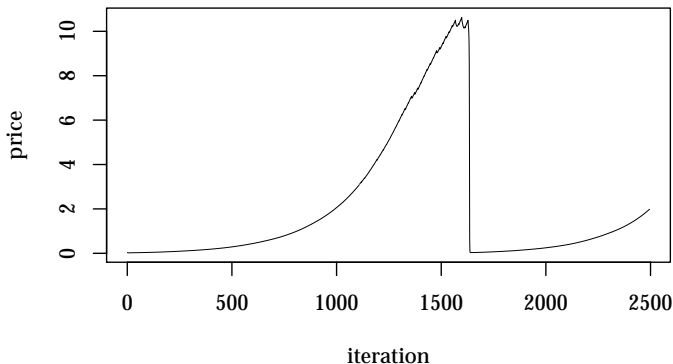


Figure: Price dynamics. Micro-failure striking every $\tau = 15$ periods.

- 1 Context and proposal
- 2 Methodological perspective
- 3 The model
- 4 Simulation and results
- 5 Concluding remarks and conceivable extension**

1 Analysis of individual wealth-shares

- strong market selection mechanism
- single- and multiple-survivor equilibria
- riskier investment functions globally dominate

2 Analysis of transitional price dynamics

- emergence of bubble-and-bust cycles
- robust to the introduction of fundamentalist and trend-contrarian

Our framework can be extended in a number of directions:

- multiple risky assets
- dynamic population
- more realistic traders' behaviour (prospect theory, herding) and learning (genetic algorithms, classifier systems)

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