Wealth-driven asymptotic survival in a financial market with demand shocks

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Outline

Context and motivation

Friedman *'as if'* argument (1953)

- economic agents can be described *as if* they were fully rational
- non-rational agents would get wiped out of the market

Empirical evidence

- trade occurs
- excess volatility
- market sentiment

Heterogeneous Agent Models (HAMs)

- analytical investigation of long-run dynamics
- incorporate various degrees of bounded rationality

Context and motivation

traditionally HAMs

- feedback mechanisms from realised market outcomes
  - e.g. fundamentalist vs. chartist (Brock and Hommes)
- long-run analysis within a notion of equilibrium
  - i.e. actions are fixed at the equilibrium
- *'deterministic skeleton'* approach

what happens if one introduces 'equilibrium' demand shocks?

Black (1986) JoF

noise traders: *"trading on noise as if it were information"*

- noisy channel
- investor sentiment
- response to pseudo-signals
- popular models, financial gurus

Context and motivation

De Long, Shleifer, Summers and Waldmann (1990) JPE

- *unsophisticated investors* vs. rational arbitrageurs
- effect on price but fixed wealth
- noise traders introduce market risk that limit arbitrage
- noise traders may well earn higher expected returns than arbitrageurs

De Long, Shleifer, Summers and Waldmann (1991) JoB

- endogenous wealth but no price effect of noise traders
- perceived mispricing makes noise traders bear more risk than arbitrageurs with similar preferences

Context and motivation

Our contribution

- constant vs. stochastic portfolio strategies
- wealth-driven market selection
- conditions for survival and dominance

Main findings

- trade-off between portfolio riskiness and variability
- long-run heterogeneity
- non-trivial price-wealth dynamics (e.g. volatility clustering)

Outline

The model

Consider a stylised financial market (no consumption)

- trade takes place in discrete time t
- risk-free bond
  - perfectly elastic supply
  - constant r_f return
  - price normalised to 1 (*numéraire*)
- long-lived risky security
  - positive dividend $d_t = d_{t-1}(1+g),\ g > r_f$
  - unitary constant supply
  - market clearing price p_t

At each time step t trader n

- invests a fraction $x_{n,t}$ of her wealth $w_{n,t}$ into the risky security
- residually invests $(1 - x_{n,t}) \cdot w_{n,t}$ into the bond

Trader n wealth $w_{n,t}$ equals the current market value of her portfolio

Definitions

- net rate of return

$$r_t := \frac{p_t}{p_{t-1}} - 1$$

- dividend yield of the risky asset

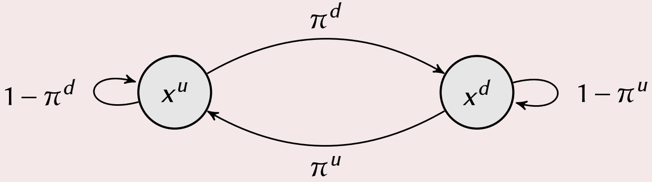
$$e_t := \frac{d_t}{p_{t-1}} = e_{t-1} \frac{1+g}{1+r_{t-1}}$$

- wealth share of agent n

$$\varphi_{n,t} := \frac{w_{n,t}}{\sum_n w_{n,t}}$$

Assumption

- constant trader always invests $\overline{x} \in (0, 1)$
- stochastic trader invests according to Markov process $\{\mathcal{X}_t,\ t \in \mathbb{N}\}$



$$\pi^u > 0 \qquad \pi^d > 0 \qquad 0 < x^d < x^u < 1$$

Laws of motion of the economy

$$\mathcal{F}_{x_{t-1},x_t} : \mathcal{D} \to \mathcal{D}, \qquad \mathcal{D} = \Delta \times (-1, +\infty) \times \mathbb{R}_+$$

$$\begin{cases} \varphi_t = \varphi_{t-1} \dfrac{1 + x_{t-1}(r_t + e_t)}{1 + (r_t + e_t)[\varphi_{t-1}x_{t-1} + (1 - \varphi_{t-1})\overline{x}]} \\ r_t = \dfrac{\varphi_{t-1}[x_t(1 + e_t x_{t-1}) - x_{t-1}] + (1 - \varphi_{t-1})e_t\overline{x}^2}{\varphi_{t-1}x_{t-1}(1 - x_t) + (1 - \varphi_{t-1})\overline{x}(1 - \overline{x})} \\ e_t = e_{t-1}\dfrac{1 + g}{1 + r_{t-1}} \end{cases}$$

where

$$\varphi_t = \frac{\text{wealth of stochastic trader}}{\text{total wealth}} \in [0, 1]$$

Outline

Terminology

- trader n is said to *survive* on $\{x_t\}_{t=0}^{\infty}$ if $\limsup_{t\to\infty} \varphi_{n,t} > 0$
- trader n is said to *vanish* on $\{x_t\}_{t=0}^{\infty}$ if $\limsup_{t\to\infty} \varphi_{n,t} = 0$
- trader n is said to *dominate* on $\{x_t\}_{t=0}^{\infty}$ if $\liminf_{t\to\infty} \varphi_{n,t} = 1$

Proposition

If $(\varphi^*, r^*, e^*) \in \mathcal{D}$ is a fixed point of system $\mathcal{F}$ then either

1. the constant trader dominates (steady state $\mathfrak{C}$)
2. the stochastic trader dominates (steady state $\mathfrak{S}$)

A dominant constant trader

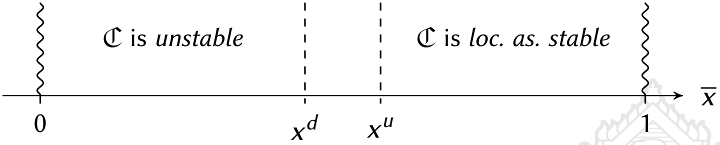
$$\varphi^{\mathfrak{C}} = 0 \qquad r^{\mathfrak{C}} = g \qquad e^{\mathfrak{C}} = g\frac{1-\overline{x}}{\overline{x}}$$

Proposition

Steady state $\mathfrak{C}$ is locally asymptotically stable if

$$\lambda^{\mathfrak{C}} = \frac{\left(\overline{x} + gx^u\right)^{\frac{\pi^u}{\pi^u+\pi^d}}\left(\overline{x} + gx^d\right)^{\frac{\pi^d}{\pi^u+\pi^d}}}{\overline{x}(1+g)} < 1$$

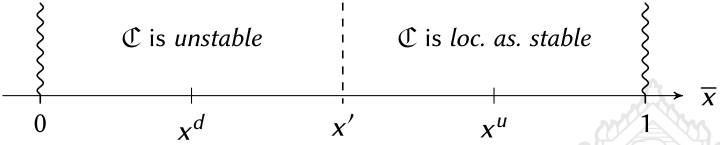
A dominant constant trader



Sufficient conditions

- $\overline{x} \geq x^u \implies \mathfrak{C}$ is locally asymptotically stable
- $\overline{x} \leq x^d \implies \mathfrak{C}$ is unstable

A dominant constant trader



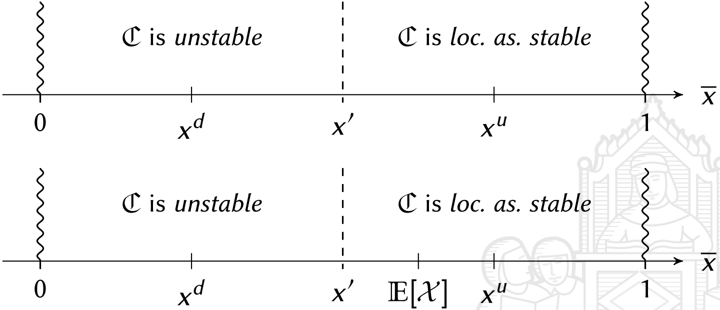
Proposition

$\exists!\ x' \in (0,1)$ such that

- $\forall\ \overline{x} > x'$, $\mathfrak{C}$ is locally asymptotically stable
- $\forall\ \overline{x} < x'$, $\mathfrak{C}$ is unstable

Moreover $x^d < x' < x^u$

A dominant constant trader



Special case

$$\pi^u = \pi^d \implies x' = \frac{x^u + x^d}{2} - h(g)$$

$$h(0) = 0$$
$$h(g) > 0$$
$$h'(g) > 0$$

Numerical result

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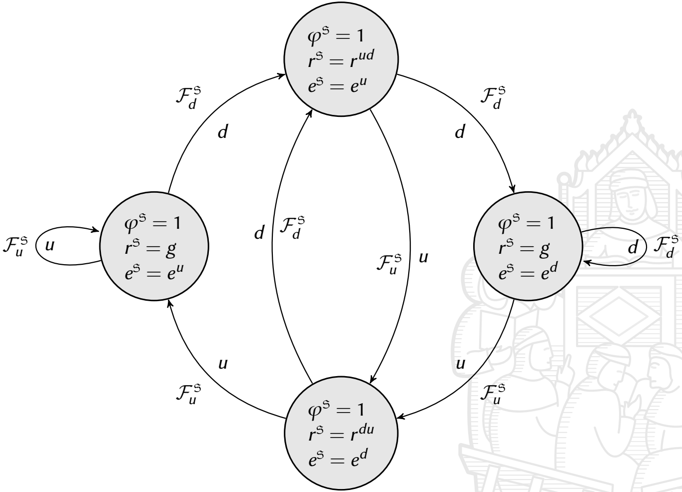


It holds $r^{ud} < g < r^{du}$ and $e^u < e^d$. Moreover, *'usually'* $r^{ud} < 0$

A dominant stochastic trader

Proposition

Attractor $\mathfrak{S}$ is an irreducible, time-homogeneous Markov chain

$$\tilde{\mathcal{P}} = \begin{array}{c} (x^u,x^u) \\ (x^u,x^d) \\ (x^d,x^u) \\ (x^d,x^d) \end{array} \overset{\begin{array}{cccc} (x^u,x^u) & (x^u,x^d) & (x^d,x^u) & (x^d,x^d) \end{array}}{\begin{pmatrix} 1-\pi^d & \pi^d & 0 & 0 \\ 0 & 0 & 1-\pi^u & \pi^u \\ 1-\pi^d & \pi^d & 0 & 0 \\ 0 & 0 & 1-\pi^u & \pi^u \end{pmatrix}}$$

Corollary

Since all states are positive-recurrent, there exists a unique invariant distribution

$$\tilde{\boldsymbol{\pi}} = \left[\frac{\pi^u(1-\pi^d)}{\pi^u+\pi^d}, \frac{\pi^u\pi^d}{\pi^u+\pi^d}, \frac{\pi^u\pi^d}{\pi^u+\pi^d}, \frac{\pi^d(1-\pi^u)}{\pi^u+\pi^d} \right]$$

A dominant stochastic trader

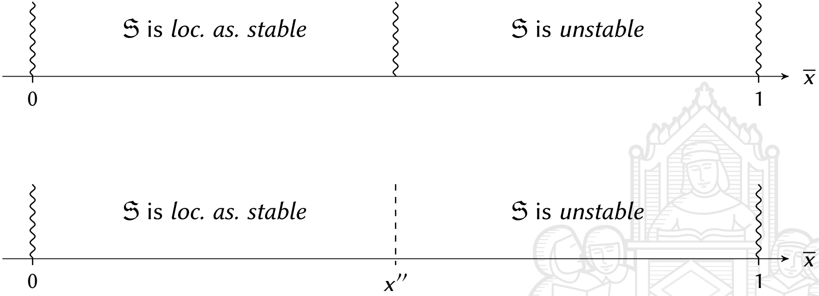
Proposition

Steady state $\mathfrak{S}$ is locally asymptotically stable if

$$\lambda^{\mathfrak{S}} = \frac{1}{1+g}\left[1+\frac{g\bar{x}}{x^u}\right]^{\frac{\pi^u(1-\pi^d)}{\pi^u+\pi^d}}$$

$$\cdot\left[\frac{\bar{x}\left[g(1-x^u)-(x^u-x^d)\right]+x^u(1-x^d)}{x^u(1-x^u)}\cdot\frac{\bar{x}\left[g(1-x^d)+(x^u-x^d)\right]+x^d(1-x^u)}{x^d(1-x^d)}\right]^{\frac{\pi^u\pi^d}{\pi^u+\pi^d}}$$

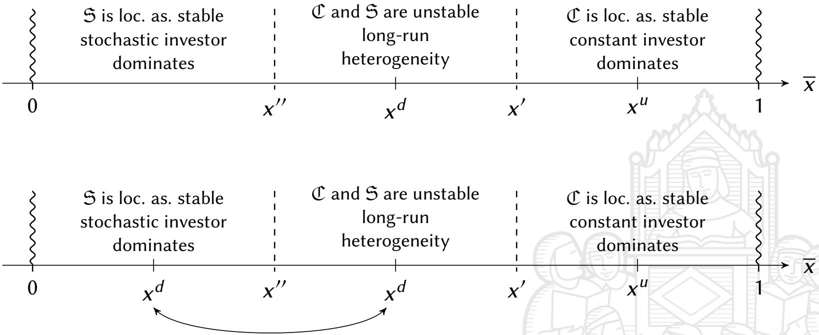
$$\cdot\left[1+\frac{g\bar{x}}{x^d}\right]^{\frac{\pi^d(1-\pi^u)}{\pi^u+\pi^d}} < 1$$



Sufficient conditions

- if $\overline{x} \leq x^d$ and $g \geq \dfrac{x^u - x^d}{1 - x^u}$ with at least one strict inequality sign, then steady state $\mathfrak{S}$ is locally asymptotically stable
- if $\overline{x} \geq x''$ and g

Long-run heterogeneity

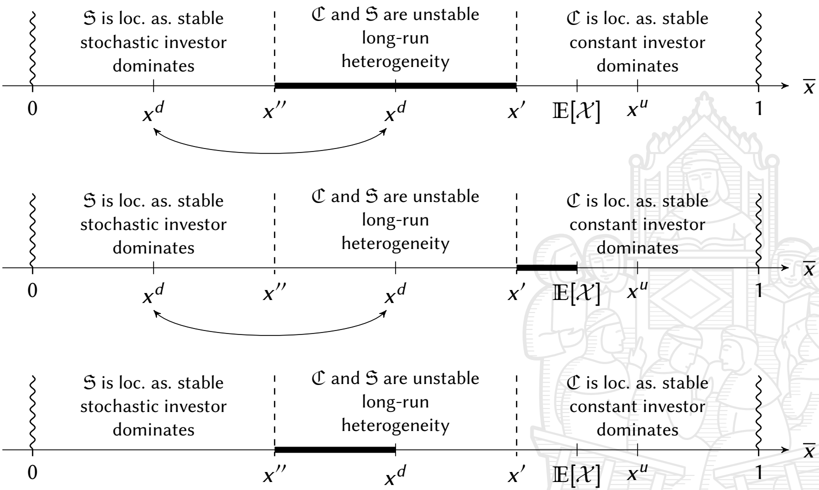


Proposition

$\exists\ \hat{g} > 0$ such that $\forall\ g < \hat{g}$ it holds $x'' < x^d < x'$. In particular

$$\hat{g} = \frac{x^u - x^d}{\ }$$

Long-run heterogeneity

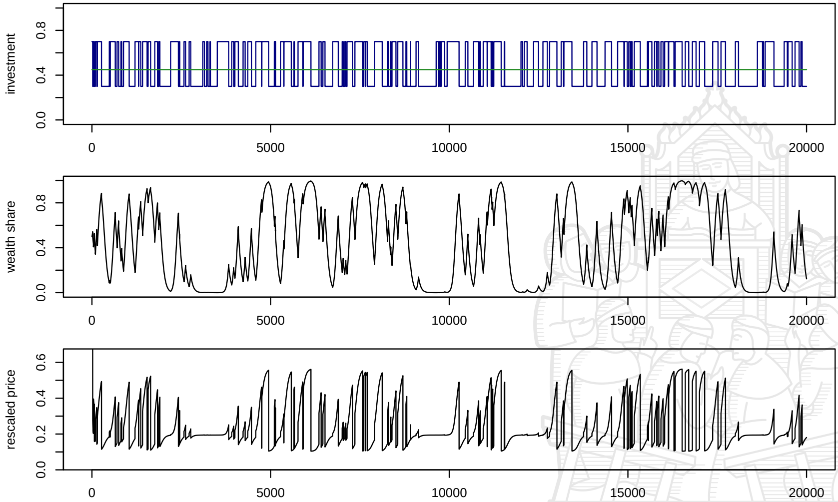


Outline

Initialisation

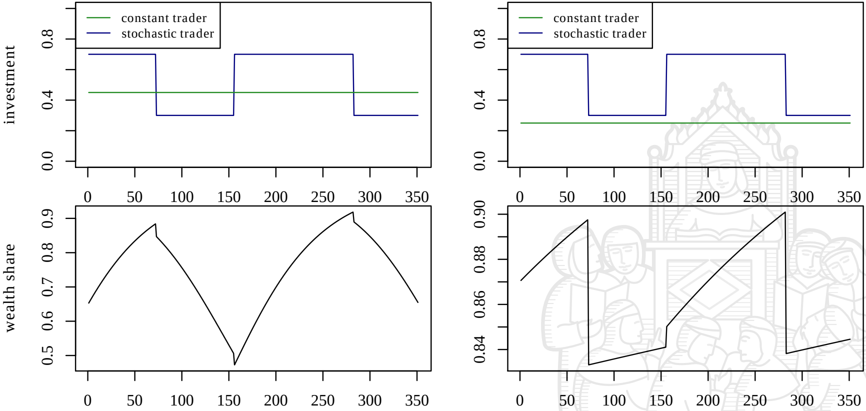
Description	Variable	Value
dividend rate of growth	g	0.05
constant investment	$\overline{x}$	0.45
stochastic investment up	x^u	0.7
stochastic investment down	x^d	0.3
probability down when up	π^d	0.01
probability up when down	π^u	0.01
initial wealth share	φ_0	0.5
initial return	r_0	0.0
initial yield	e_0	0.01

Simulation



investment
0.0
0.4
0.8
0
5000
10000
15000
20000
wealth share
0.0
0.4
0.8
0
5000
10000
15000
20000
rescaled price
0.0
0.2
0.4
0.6
0
5000
10000
15000
20000

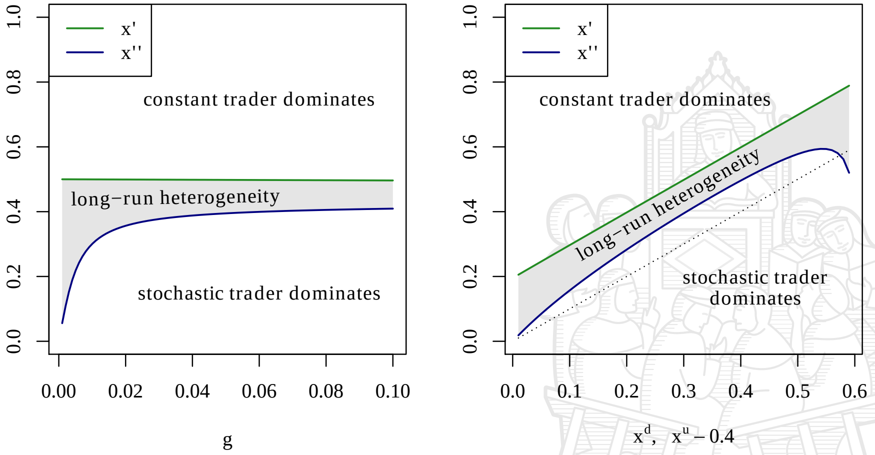
A tale of two regimes



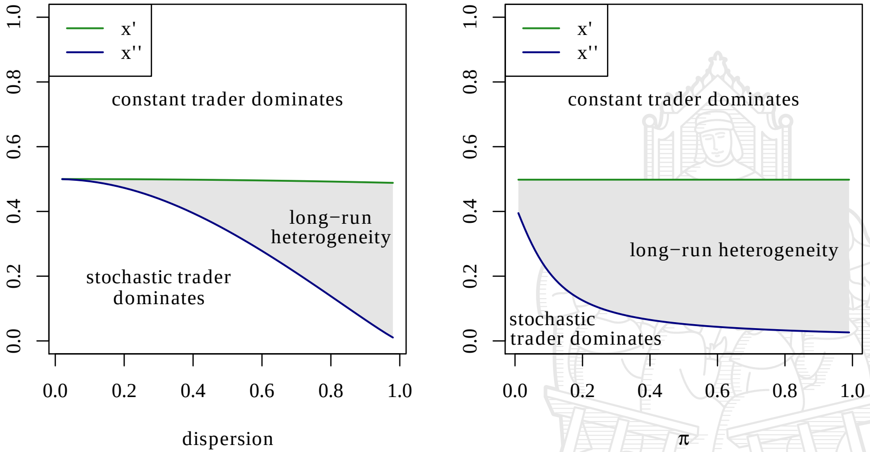
1st regime: $g = 0.05$ and $\overline{x} = 0.45$

2nd regime: $g = 0.005$ and $\overline{x} = 0.25$

Sensitivity analysis



Sensitivity analysis



Outline

Concluding remarks

Main findings

- trade-off between portfolio riskiness and variability
- $\exists$ intrinsic penalty in adopting stochastic strategy
- a constant trader can dominate a stochastic trader with an on-average safer portfolio
- a constant trader can invade a stochastic trader with an always-strictly-safer portfolio
- the stochastic trader (fire-)sells at low price and buys at high price
- generic long-run heterogeneity
- endogenous fluctuations of price and wealth
- volatility clustering
- loss of generality in adopting the *deterministic skeleton* approach

Concluding remarks

Conceivable extensions

- arbitrary number of traders
- arbitrary number of risky securities
- real sector, credit system, monetary authority
- endogenise g and/or r_f

Hope you slept comfortably

Thank you very much!

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