Wealth-driven asymptotic survival in a financial market with demand shocks

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Sant'Anna
Scuola Universitaria Superiore Pisa

Outline

Context and motivation

Friedman *'as if'* argument (1953)

- economic agents can be described *as if* they were fully rational
- non-rational agents would get wiped out of the market

Empirical evidence

- trade occurs
- excess volatility
- market sentiment

Heterogeneous Agent Models (HAMs)

- analytical investigation of long-run dynamics
- incorporate various degrees of bounded rationality

Context and motivation (cont'd)

traditionally HAMs

- feedback mechanisms from realised market outcomes
- e.g. fundamentalist vs. chartist (Brock and Hommes)
- long-run analysis within a notion of equilibrium
- i.e. actions are fixed at the equilibrium

what happens if one introduces 'equilibrium' demand shocks?

Noise traders

"trading on noise as if it were information" Black (1986)

- noisy channel
- investor sentiment
- response to pseudo-signals
- popular models, financial gurus

Context and motivation (cont'd)

Our contribution

- constant vs. stochastic portfolio strategies
- wealth-driven market selection
- conditions for survival and dominance

Main findings

- trade-off between portfolio riskiness and variability
- long-run heterogeneity
- non-trivial price-wealth dynamics (e.g. volatility clustering)

Outline

The model

Consider a stylised financial market (no consumption)

- trade takes place in discrete time t
- risk-free bond
  - perfectly elastic supply
  - constant r_f return
  - price normalised to 1 (*numéraire*)
- long-lived risky security
  - positive dividend $d_t = d_{t-1}(1+g)$, $g > r_f$
  - unitary constant supply
  - market clearing price p_t

At each time step t trader n

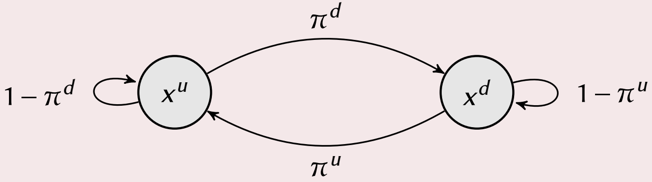
- invests a fraction $x_{n,t}$ of her wealth $w_{n,t}$ into the risky security
- residually invests $(1 - x_{n,t}) \cdot w_{n,t}$ into the bond

Trader n wealth $w_{n,t}$ equals the current market value of her portfolio

Traders' behaviour

Assumption

- constant trader always invests $\overline{x} \in (0, 1)$
- stochastic trader invests according to Markov process $\{\mathcal{X}_t,\ t \in \mathbb{N}\}$



$$\pi^u > 0 \qquad\qquad 0 < x^d < x^u < 1$$
$$\pi^d > 0$$

Random dynamical system

$$\mathcal{F}_{x_{t-1},x_t} : \mathcal{D} \to \mathcal{D}, \qquad \mathcal{D} = \Delta \times (-1, +\infty) \times \mathbb{R}_+$$

$$\begin{cases} \varphi_t = \varphi_{t-1} \dfrac{1 + x_{t-1}(r_t + e_t)}{1 + (r_t + e_t)\left[\varphi_{t-1} x_{t-1} + (1 - \varphi_{t-1})\bar{x}\right]} \\[2ex] r_t = \dfrac{\varphi_{t-1}\left[x_t(1 + e_t x_{t-1}) - x_{t-1}\right] + (1 - \varphi_{t-1}) e_t \bar{x}^2}{\varphi_{t-1} x_{t-1}(1 - x_t) + (1 - \varphi_{t-1})\bar{x}(1 - \bar{x})} \\[2ex] e_t = e_{t-1} \dfrac{1 + g}{1 + r_{t-1}} \end{cases}$$

where φ_t denotes the wealth share of the stochastic trader

$$\varphi_t = \frac{\text{wealth of stochastic trader}}{\text{total wealth}} \in [0, 1]$$

Outline

Long-run outcomes

Terminology

- trader n is said to *survive* on $\{x_t\}_{t=0}^{\infty}$ if $\limsup_{t\to\infty} \varphi_{n,t} > 0$
- trader n is said to *vanish* on $\{x_t\}_{t=0}^{\infty}$ if $\limsup_{t\to\infty} \varphi_{n,t} = 0$
- trader n is said to *dominate* on $\{x_t\}_{t=0}^{\infty}$ if $\liminf_{t\to\infty} \varphi_{n,t} = 1$

Proposition

If $(\varphi^*, r^*, e^*) \in \mathcal{D}$ is a fixed point of system $\mathcal{F}$ then either

1. the constant trader dominates (steady state $\mathfrak{C}$)
2. the stochastic trader dominates (steady state $\mathfrak{S}$)

A dominant constant trader

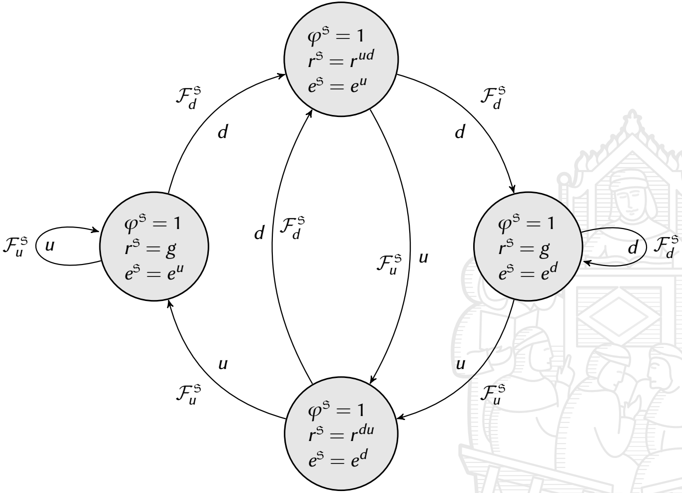
$$\varphi^{\mathfrak{C}} = 0 \qquad r^{\mathfrak{C}} = g \qquad e^{\mathfrak{C}} = g\frac{1-\overline{x}}{\overline{x}}$$

Proposition

Steady state $\mathfrak{C}$ is locally asymptotically stable if

$$\lambda^{\mathfrak{C}} = \frac{\left(\overline{x} + g x^u\right)^{\frac{\pi^u}{\pi^u+\pi^d}} \left(\overline{x} + g x^d\right)^{\frac{\pi^d}{\pi^u+\pi^d}}}{\overline{x}(1+g)} < 1$$

A dominant stochastic trader



It holds $r^{ud} < g < r^{du}$ and $e^u < e^d$. Moreover, *'usually'* $r^{ud} < 0$

A dominant stochastic trader (cont'd)

Proposition

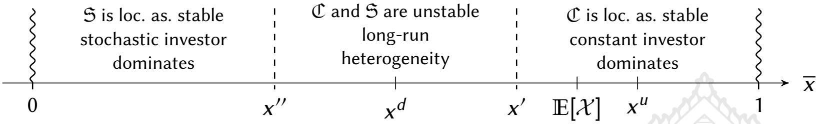
Steady state $\mathfrak{S}$ is locally asymptotically stable if

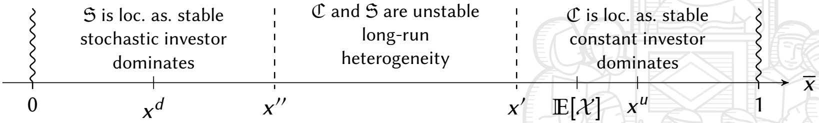
$$\lambda^{\mathfrak{S}} = \frac{1}{1+g}\left[1+\frac{g\bar{x}}{x^u}\right]^{\frac{\pi^u(1-\pi^d)}{\pi^u+\pi^d}}$$

$$\cdot\left[\frac{\bar{x}\left[g(1-x^u)-(x^u-x^d)\right]+x^u(1-x^d)}{x^u(1-x^u)}\cdot\frac{\bar{x}\left[g(1-x^d)+(x^u-x^d)\right]+x^d(1-x^u)}{x^d(1-x^d)}\right]^{\frac{\pi^u\pi^d}{\pi^u+\pi^d}}$$

$$\cdot\left[1+\frac{g\bar{x}}{x^d}\right]^{\frac{\pi^d(1-\pi^u)}{\pi^u+\pi^d}} < 1$$

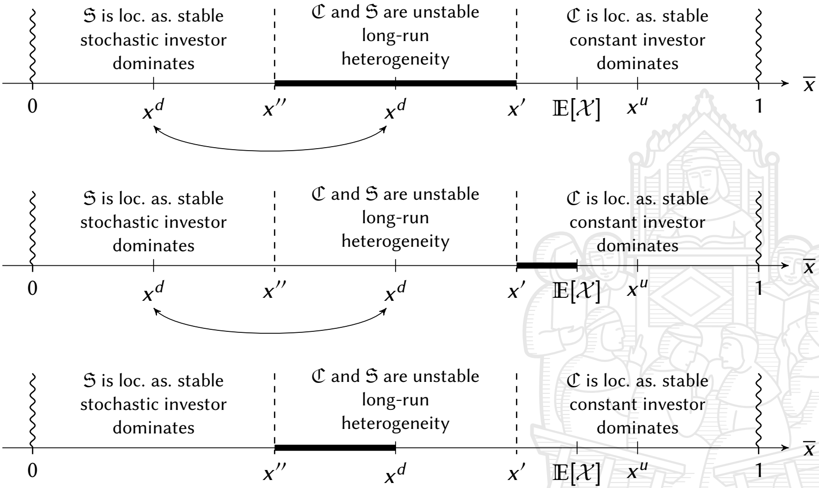
Stability regions





▸ $\mathfrak{C}$ stability analysis   ▸ $\mathfrak{S}$ stability analysis

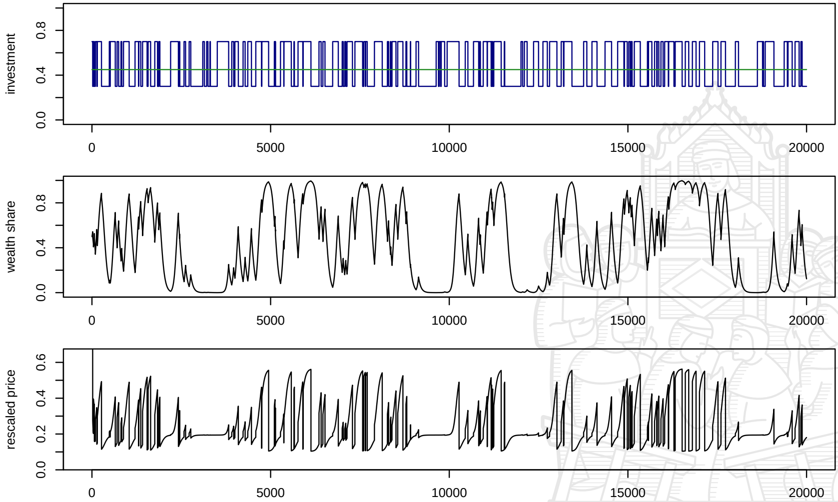
Stability regions (cont'd)



Corollary

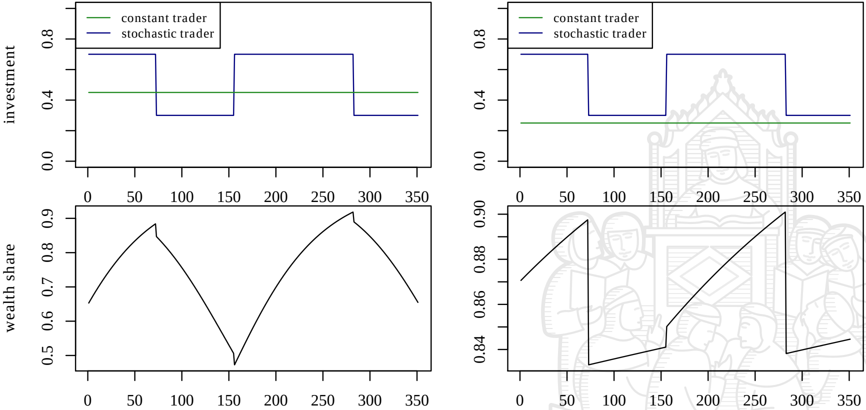
Outline

Simulation



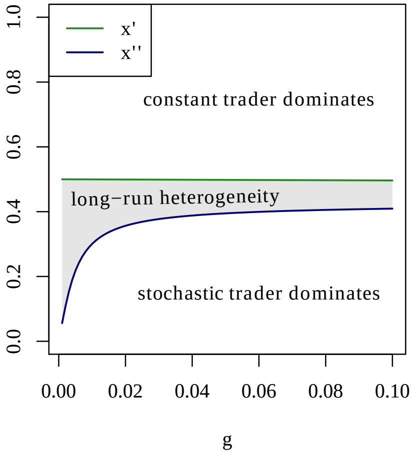
investment
0.0
0.4
0.8
0
5000
10000
15000
20000
wealth share
0.0
0.4
0.8
0
5000
10000
15000
20000
rescaled price
0.0
0.2
0.4
0.6
0
5000
10000
15000
20000

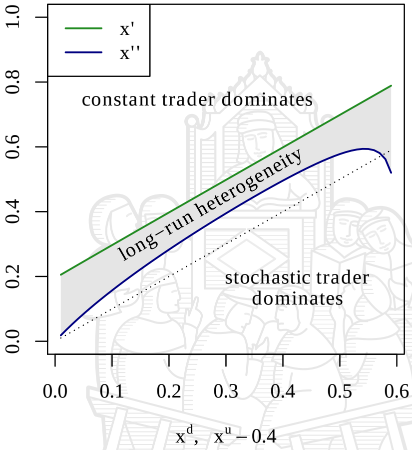
A tale of two regimes



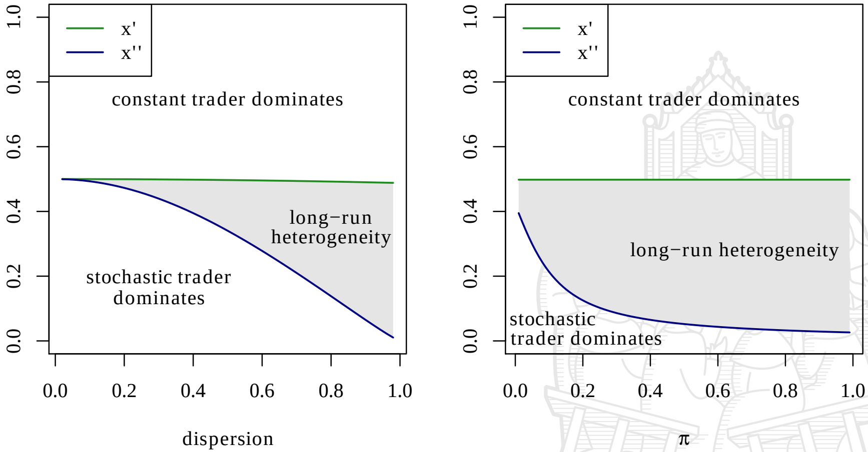
conclusions

Sensitivity analysis





Sensitivity analysis (cont'd)



Outline

Concluding remarks

Main findings

- trade-off between portfolio riskiness and variability
- ∃ intrinsic penalty in adopting stochastic strategy
- a constant trader can dominate a stochastic trader with an on-average safer portfolio
- a constant trader can invade a stochastic trader with an always-strictly-safer portfolio
- the stochastic trader (fire-)sells at low price and buys at high price
- generic long-run heterogeneity
- endogenous fluctuations of price and wealth
- volatility clustering

Concluding remarks (cont'd)

Conceivable extensions

- arbitrary number of traders
- arbitrary number of risky securities
- real sector, credit system, monetary authority
- endogenise g and/or r_f

Hope you slept comfortably

Thank you very much!

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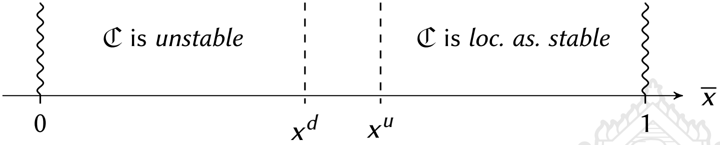
. . . and props to the European Project PIOF-GA-2011-300637 MSAEO for financial support

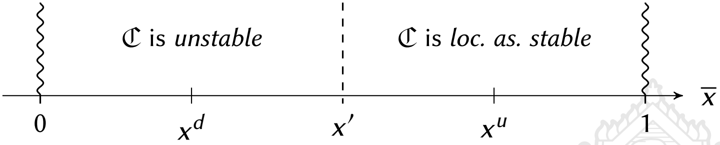
$\mathfrak{C}$ stability analysis



Sufficient conditions

- $\overline{x} \geq x^u \implies$ $\mathfrak{C}$ is locally asymptotically stable
- $\overline{x} \leq x^d \implies$ $\mathfrak{C}$ is unstable

$\mathfrak{C}$ stability analysis



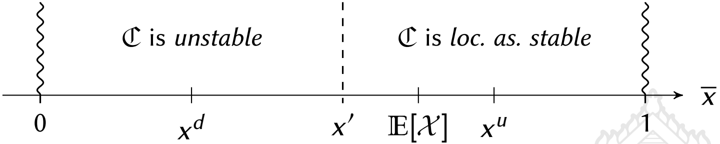
Proposition

$\exists!\ x' \in (0,1)$ such that

- $\forall\ \overline{x} > x'$, $\mathfrak{C}$ is locally asymptotically stable
- $\forall\ \overline{x} < x'$, $\mathfrak{C}$ is unstable

Moreover $x^d < x' < x^u$

$\mathfrak{C}$ stability analysis



Special case

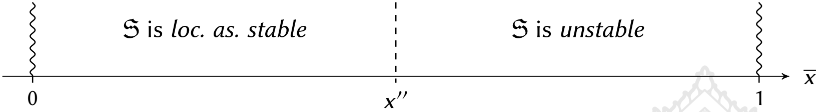
$$\pi^u = \pi^d \quad \Longrightarrow \quad x' = \frac{x^u + x^d}{2} - h(g)$$

$$h(0) = 0$$
$$h(g) > 0$$
$$h'(g) > 0$$

Numerical result

$x' \leq \mathbb{E}[\mathcal{X}]$ even if $\pi^u \neq \pi^d$

$\mathfrak{S}$ stability analysis

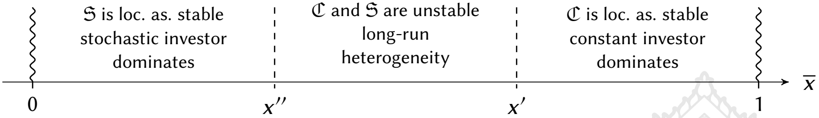


Numerical result

$\exists!\ x'' \in (0,1)$ such that

- $\forall\ \overline{x} < x''$, $\mathfrak{S}$ is locally asymptotically stable
- $\forall\ \overline{x} > x''$, $\mathfrak{S}$ is unstable

$\mathfrak{S}$ stability analysis

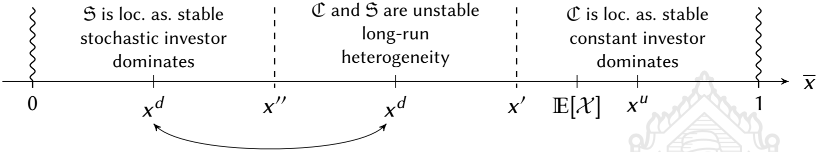


There *generically* exists a non-degenerate interval (x'', x') such that both $\mathfrak{C}$ and $\mathfrak{S}$ are unstable $\forall\ \overline{x} \in (x'', x')$

Proposition

$\exists\ \hat{g} > 0$ such that $\forall\ g < \hat{g}$ it holds $x'' < x^d < x'$. In particular

$$\hat{g} = \frac{x^u - x^d}{1 - x^u}$$



Numerical result

A numerical inspection of the parameter space reveals that $x'' < x'$

- $\forall\ x^u, x^d, \pi^u, \pi^d \in \{0.01, 0.02, \ldots, 0.99\}$ such that $x^d < x^u$
- $\forall\ g = \hat{g} \cdot 10^k, k \in \mathbb{N}_+$ such that $g \leq 10$

back