

Wealth-driven asymptotic survival in a financial market with demand shocks

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- 1 **Context and motivation**
- 2 The model
- 3 Long-run outcomes
- 4 Simulation
- 5 Concluding remarks



Black (1986) JoF

noise traders: *“trading on noise as if it were information”*

Noise traders vs. rational arbitrageurs

De Long et al. (1990) JPE price impact, fixed wealth

- market risk introduced by noise traders prevents short horizon arbitrageurs from betting against them
- noise traders may earn higher expected returns

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- perceived mispricing makes noise traders bear more risk than arbitrageurs with similar preferences
- higher growth of wealth

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Our contribution

- analytical HAM with *both* endogenous price and wealth
- fixed-mix vs. stochastic portfolio strategies
- wealth-driven market selection
- sufficient conditions for survival and dominance of both strategies

Main findings

- trade-off between portfolio riskiness and investment variability
- generic long-run heterogeneity
- non-trivial price-wealth dynamics (e.g. volatility clustering)
- loss of generality in adopting the '*deterministic skeleton*' approach

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The model

Consider a stylised financial market (no consumption)

- trade takes place in discrete time t
- risk-free bond
 - perfectly elastic supply
 - constant r_f return
 - price normalised to 1 (*numéraire*)
- long-lived risky security
 - positive dividend $d_t = d_{t-1}(1 + g)$, $g > r_f$
 - unitary constant supply
 - market clearing price p_t

At each time step t trader n

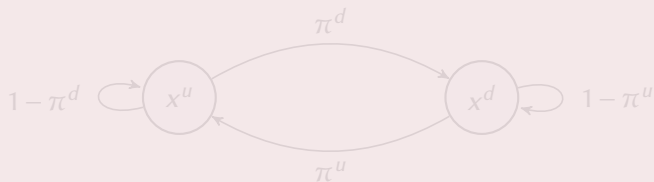
- invests a fraction $x_{n,t}$ of her wealth $w_{n,t}$ into the risky security
- residually invests $(1 - x_{n,t}) \cdot w_{n,t}$ into the bond

Trader n wealth $w_{n,t}$ equals the current market value of her portfolio



Assumption

- constant trader always invests $\bar{x} \in (0, 1)$
- stochastic trader invests according to Markov process $\{\mathcal{X}_t, t \in \mathbb{N}\}$



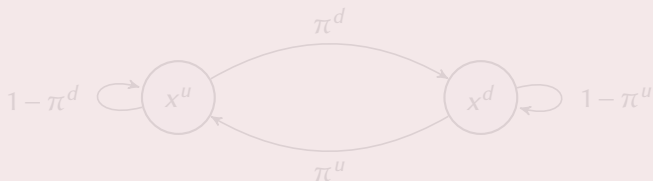
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$$\pi^d > 0$$

$$0 < x^d < x^u < 1$$

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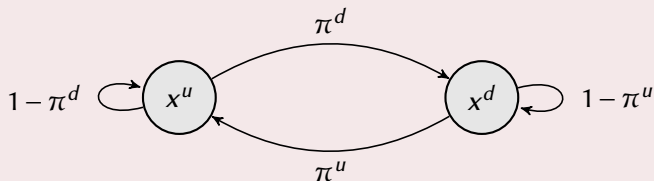
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Traders' behaviour

Assumption

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Example

- a generic trader commits to a fixed-mix strategy
- a liquidity shock occurs with $\pi > 0$

Two cases:

low-risk portfolio (constant trader)

use bond as collateral (no recomposition)

high-risk portfolio (stochastic trader)

sell stock, purchase bond and use as collateral (fire sale)

- should a common external shock simultaneously hit both agent types, the high-risk trader might be unable to avoid a sudden portfolio recomposition, while the low-risk trader can

Laws of motion of the economy

$$\mathcal{F}_{x_{t-1}, x_t} : \mathcal{D} \rightarrow \mathcal{D}, \quad \mathcal{D} = \Delta \times (-1, +\infty) \times \mathbb{R}_{++}$$

$$\left\{ \begin{array}{l} \varphi_t = \varphi_{t-1} \frac{1 + x_{t-1}(r_t + e_t)}{1 + (r_t + e_t)[\varphi_{t-1}x_{t-1} + (1 - \varphi_{t-1})\bar{x}]} \\ r_t = \frac{\varphi_{t-1}[x_t(1 + e_t x_{t-1}) - x_{t-1}] + (1 - \varphi_{t-1})e_t \bar{x}^2}{\varphi_{t-1}x_{t-1}(1 - x_t) + (1 - \varphi_{t-1})\bar{x}(1 - \bar{x})} \\ e_t = e_{t-1} \frac{1 + g}{1 + r_{t-1}} \end{array} \right.$$

where

$$\varphi_t = \frac{\text{wealth of stochastic trader}}{\text{total wealth}} \in [0, 1]$$

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Terminology

- trader n is said to *survive* on $\{x_t\}_{t=0}^{\infty}$ if $\limsup_{t \rightarrow \infty} \varphi_{n,t} > 0$
- trader n is said to *vanish* on $\{x_t\}_{t=0}^{\infty}$ if $\limsup_{t \rightarrow \infty} \varphi_{n,t} = 0$
- trader n is said to *dominate* on $\{x_t\}_{t=0}^{\infty}$ if $\liminf_{t \rightarrow \infty} \varphi_{n,t} = 1$

Proposition

If $(\varphi^*, r^*, e^*) \in \mathcal{D}$ is a fixed point of system \mathcal{F} then either

- 1 the constant trader dominates (steady state \mathbb{C})
- 2 the stochastic trader dominates (steady state \mathbb{S})



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A dominant constant trader

$$\varphi^c = 0$$

$$r^c = g$$

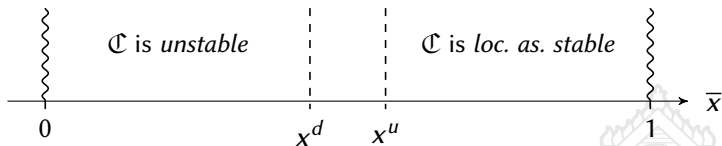
$$e^c = g \frac{1 - \bar{x}}{\bar{x}}$$

Proposition

Steady state \mathcal{C} is locally asymptotically stable if

$$\lambda^c = \frac{(\bar{x} + gx^u)^{\frac{\pi^u}{\pi^u + \pi^d}} (\bar{x} + gx^d)^{\frac{\pi^d}{\pi^u + \pi^d}}}{\bar{x}(1 + g)} < 1$$

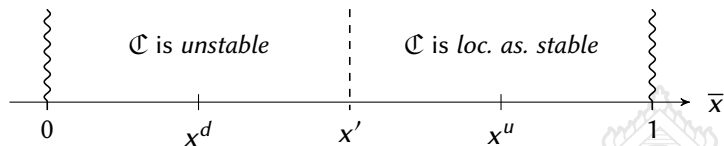
A dominant constant trader



Sufficient conditions

- $\bar{x} \geq x^u \implies \mathcal{C}$ is locally asymptotically stable
- $\bar{x} \leq x^d \implies \mathcal{C}$ is unstable

A dominant constant trader



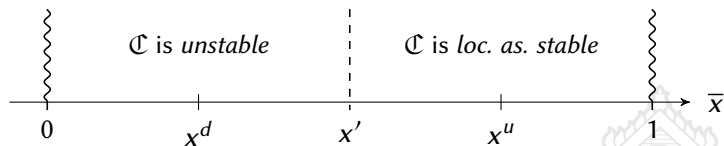
Proposition

$\exists!$ $x' \in (0, 1)$ such that

- $\forall \bar{x} > x'$, \mathcal{C} is locally asymptotically stable
- $\forall \bar{x} < x'$, \mathcal{C} is unstable

Moreover $x^d < x' < x^u$

A dominant constant trader



Special case

$$\pi^u = \pi^d \implies x' = \frac{x^u + x^d}{2} - h(g)$$

$$h(0) = 0$$

$$h(g) > 0$$

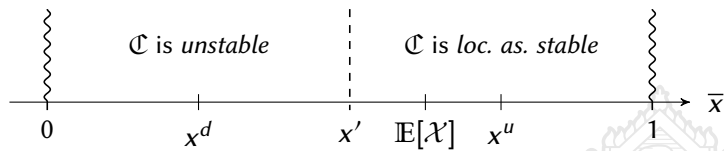
$$h'(g) > 0$$

Proposition

$x' < \mathbb{E}[\mathcal{X}]$ even if $\pi^u \neq \pi^d$



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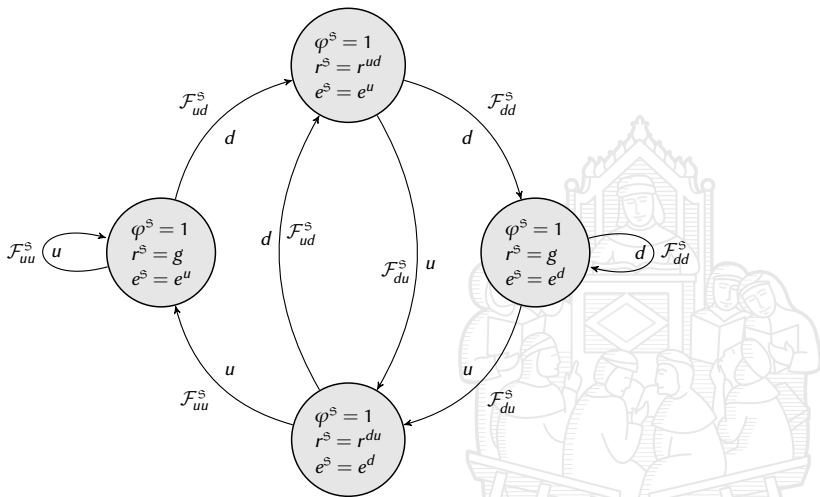
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Proposition

$$x' < \mathbb{E}[\mathcal{X}] \quad \text{even if} \quad \pi^u \neq \pi^d$$



A dominant stochastic trader



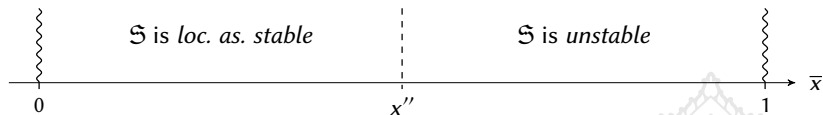
It holds $r^{ud} < g < r^{du}$ and $e^u < e^d$. Moreover, 'usually' $r^{ud} < 0$

Proposition

Steady state \mathfrak{S} is locally asymptotically stable if

$$\lambda^{\mathfrak{S}} = \frac{1}{1+g} \left[1 + \frac{g\bar{x}}{x^u} \right]^{\frac{\pi^u(1-\pi^d)}{\pi^u+\pi^d}}$$
$$\cdot \left[\frac{\bar{x} [g(1-x^u) - (x^u - x^d)] + x^u(1-x^d)}{x^u(1-x^u)} \cdot \frac{\bar{x} [g(1-x^d) + (x^u - x^d)] + x^d(1-x^u)}{x^d(1-x^d)} \right]^{\frac{\pi^u\pi^d}{\pi^u+\pi^d}}$$
$$\cdot \left[1 + \frac{g\bar{x}}{x^d} \right]^{\frac{\pi^d(1-\pi^u)}{\pi^u+\pi^d}} < 1$$

A dominant stochastic trader

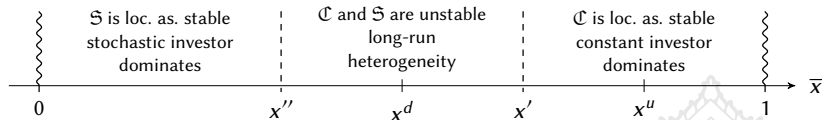


Numerical result

$\exists! x'' \in (0, 1)$ such that

- $\forall \bar{x} < x''$, \mathcal{S} is locally asymptotically stable
- $\forall \bar{x} > x''$, \mathcal{S} is unstable

Long-run heterogeneity

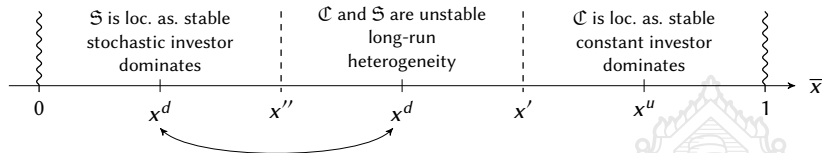


Proposition

$\exists \hat{g} > 0$ such that $\forall g < \hat{g}$ it holds $x'' < x^d < x'$. In particular

$$\hat{g} = \frac{x^u - x^d}{1 - x^u}$$

Long-run heterogeneity

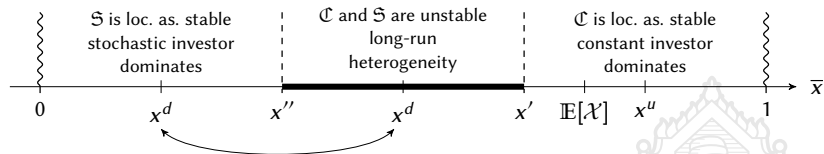


Numerical result

the relation $x'' < x'$ holds

- $\forall x^u, x^d, \pi^u, \pi^d \in \{0.01, 0.02, \dots, 0.99\}$ such that $x^d < x^u$
- $\forall g = \hat{g} \cdot 10^k, k \in \mathbb{N}_+$ such that $g \leq 10$

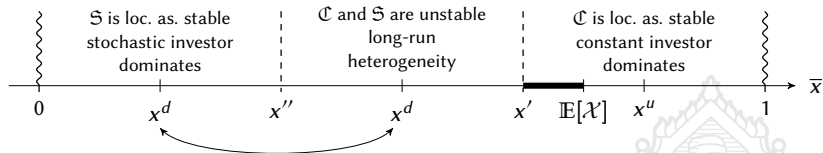
Long-run heterogeneity



Corollary

- 1 there exists a **non-degenerate** interval (x'', x') such that both \mathcal{C} and \mathcal{S} are unstable $\forall \bar{x} \in (x'', x')$
- 2 the constant trader is able to *dominate* the stochastic trader even adopting a **on-average safer** position $\bar{x} < \mathbb{E}[\mathcal{X}]$
- 3 the constant trader is able to *invade* the stochastic trader even adopting an **always strictly safer** position $\bar{x} < x^d < x^u$ when $x'' < x^d$

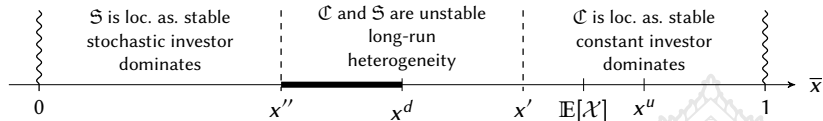
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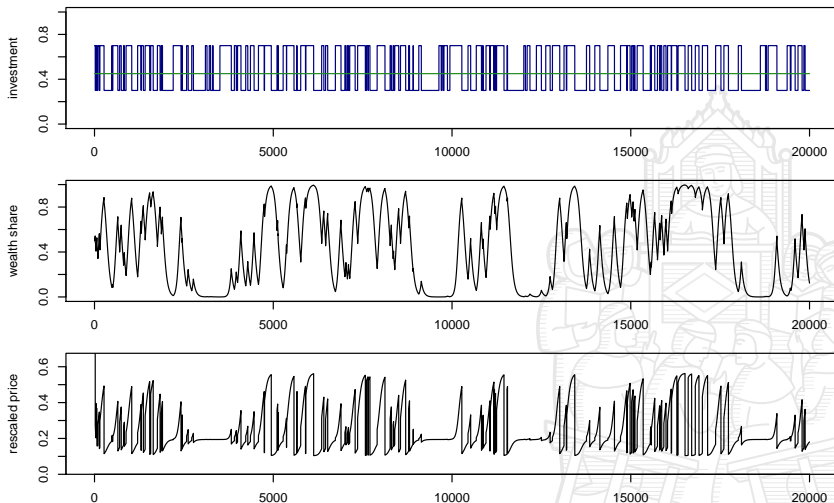
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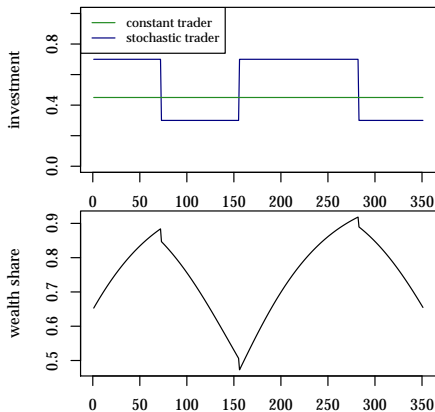


Description	Variable	Value
stochastic investment up	x^u	0.7
stochastic investment down	x^d	0.3
probability down when up	π^d	0.01
probability up when down	π^u	0.01
constant investment	\bar{x}	0.45
dividend rate of growth	g	0.05
initial wealth share	φ_0	0.5
initial return	r_0	0.0
initial yield	e_0	0.01

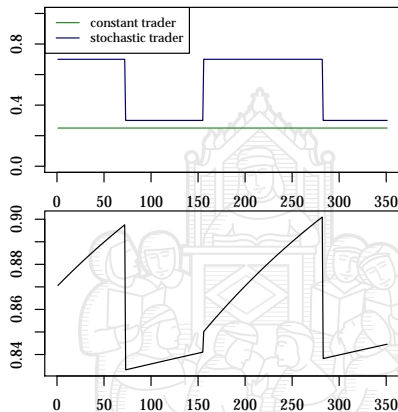
Simulation



A tale of two regimes

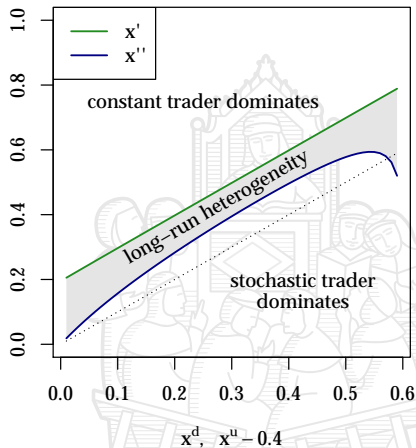
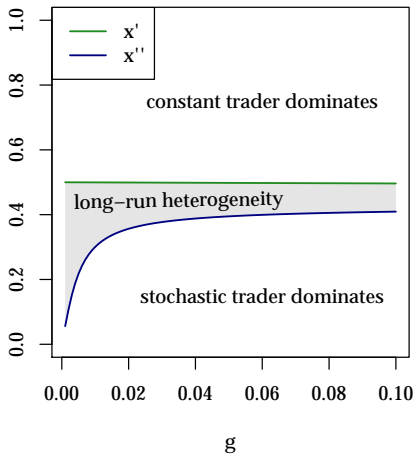


1st regime: $g = 0.05$ and $\bar{x} = 0.45$

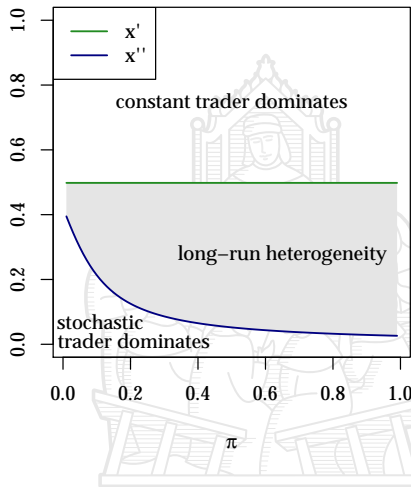
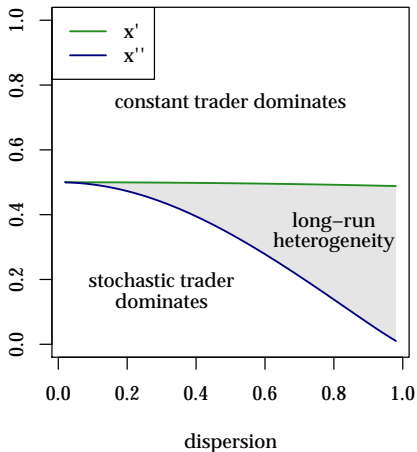


2nd regime: $g = 0.005$ and $\bar{x} = 0.25$

Sensitivity analysis



Sensitivity analysis

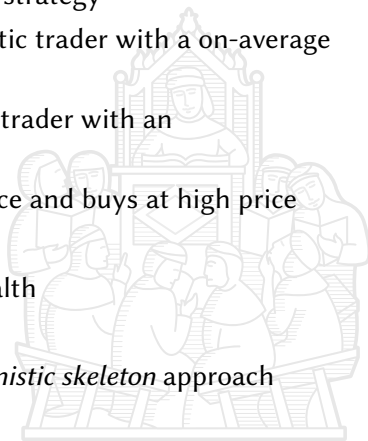


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Concluding remarks

- trade-off between portfolio riskiness and variability
- \exists intrinsic penalty in adopting stochastic strategy
- a constant trader can dominate a stochastic trader with a on-average safer portfolio
- a constant trader can invade a stochastic trader with an always-strictly-safer portfolio
- the stochastic trader (fire-)sells at low price and buys at high price
- generic long-run heterogeneity
- endogenous fluctuations of price and wealth
- volatility clustering
- loss of generality in adopting the *deterministic skeleton* approach



Hope you slept comfortably

Thank you very much!

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...and props to the European Project PEOF-GA-2011-300637 MSAEO for financial support