Bubble-and-bust dynamics under walrasian asset pricing and heterogeneous traders

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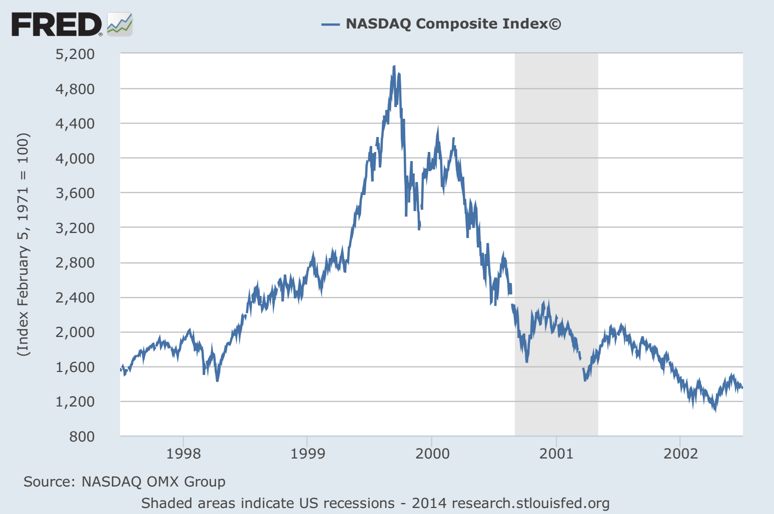
Outline

1. Context and proposal
2. Methodological perspective
3. The model
4. Simulation and results
5. Concluding remarks and conceivable extension

Financial bubbles: a neglected stylised fact



Financial bubbles (cont'd)

> *"if the reason that the price is high today is only because investors believe that the selling price will be high tomorrow – when 'fundamental' factors do not seem to justify such a price – then a bubble exists"* [Stiglitz, 1990]

> *"a sharp rise in the price of an asset or a range of assets in a continuous process, with the initial rise generating expectations of further rises and attracting new buyers – generally speculators interested in profits from trading in the asset rather than its use or earnings capacity"* [Kindleberger, 1978]

Financial bubbles (cont'd)

Our proposal

We set up a model able to yield:

- *endogenous* bubble-and-bust dynamics
- as a result of the sole interaction among *heterogeneous adaptive traders*
- highlighting booms and crashes as intrinsic features of financial markets

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Methodological perspective

Heterogeneous Agents Models

- analytical investigations of the dynamical systems representing the laws of motion of the economy
- analytical tractability often leads to restrictive simplifying assumptions
- focus on asymptotic properties

Agent-Based Models

- computational (numerical) study of economies modelled as evolving systems of interacting agents
- complex behaviour specifications
- keep track of the whole dynamics

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The model

Following Anufriev et al. (2012), consider a pure-exchange economy:

- N heterogeneous traders (index $\mathcal{N} = \{1, \ldots, n, \ldots, N\}$);
- L long-lived risky securities (index $\mathcal{L} = \{1, \ldots, \ell, \ldots, L\}$);
- a riskless bond;
- time is discrete (index $t \in \mathcal{T}$);
- risky securities, present in fixed amount, have ex-dividend price p_t^ℓ and pay random dividend d_t^ℓ at the end of each period;
- the bond, inelastically supplied, has price normalized to 1 (*numéraire*) and yields $r_f > 0$ in every t;
- trader wealth $W_{n,t}$ equals the market value of the portfolio he holds;
- fundamentals are proxied by the dividend yield $e_t^\ell = \frac{d_t^\ell}{p_{t-1}^\ell}, \forall \ell \in \mathcal{L}$.

Trader behaviour

At the beginning of each time step, trader n invests a share $x_{n,t}^{\ell}$ of his wealth in security ℓ; the decision is made according to the information set

$$\mathcal{I}_t = \{p_\tau^1, \dots, p_\tau^L;\ d_\tau^1, \dots, d_\tau^L \mid \tau < t\}$$

that is common knowledge, and to trader-specific investment function

$$f_n : \mathbb{R}^{\tau \times L} \longrightarrow \mathbb{R}^L \quad \text{such that} \quad \boldsymbol{x}_{n,t} = f_{n,t}(\mathcal{I}_t)$$

that is independent on wealth, as we assume overall CRRA attitude.

At every time step, each trader faces an optimisation problem of the form:

$$\max_{\boldsymbol{x}_{n,t}} \ \mathbb{E}\left[\frac{W_{n,t}^{1-\gamma_n} - 1}{1-\gamma_n}\right]$$

s.t.

$$W_{n,t} = W_{n,t-1} \cdot \left[x_{n,t-1}^0 \cdot (1 + r_f) + \sum_{\ell=1}^{L} x_{n,t-1}^{\ell} \cdot \left(\frac{p_t^{\ell}}{p_{t-1}^{\ell}} + e_t^{\ell}\right)\right]$$

where $\gamma_n > 0$ denotes the risk-aversion coefficient.

Trader expectations

We assume the trader forms expectations about future price returns and their (co)variances by means of EWMA estimators over the information set previously defined:

$$\hat{\rho}^{\ell}_{n,t} = \lambda_n \cdot \sum_{\tau=0}^{\infty} (1-\lambda_n)^{\tau} \cdot \rho^{\ell}_{t-\tau-1}$$

$$\hat{\sigma}^{\ell,h}_{\rho,n,t} = \lambda_n \cdot \sum_{\tau=0}^{\infty} (1-\lambda_n)^{\tau} \cdot \left[\rho^{\ell}_{t-\tau-1} - \hat{\rho}^{\ell}_{n,t-\tau-1}\right] \cdot \left[\rho^{h}_{t-\tau-1} - \hat{\rho}^{h}_{n,t-\tau-1}\right]$$

where $\rho^{\ell}_t = \frac{p^{\ell}_t}{p^{\ell}_{t-1}} - 1$ is the price return of security ℓ between $t-1$ and t. The memory decay factor $\lambda_n \in (0,1)$ captures the way relative weights are distributed across more recent and older observations.

Trader investment function

We adopt the same mean-variance approximation of the optimal investment function proposed in Chiarella and He (2001):

$$\boldsymbol{x}_{n,t} = \frac{1}{\gamma_n} \cdot \mathbb{C}_{n,t}^{-1} \cdot \left[\mathbb{E}_{n,t} - r_f \cdot \mathbf{1} \right]$$

$$\mathbb{E}_{n,t}^{\ell} = \bar{e}^{\ell} + d_n \cdot \hat{\rho}_{n,t}^{\ell}$$
$$\mathbb{C}_{n,t}^{\ell,h} = \hat{\sigma}_{\rho,n,t}^{\ell,h} + \sigma_e^{\ell,h}$$

where $\mathbb{E}_{n,t}$ and $\mathbb{C}_{n,t}$ are, respectively, the vector of expected total returns and the expected variance-covariance matrix and d_n is a behavioural parameter:

- $d_n = 0$ trader n is a fundamentalist
- $d_n > 0$ trader n is a trend-chaser
- $d_n < 0$ trader n is a trend-contrarian

Assumption

e_t^{ℓ} is drown at each time step from a L-dimensional known probability distribution with mean $\bar{\boldsymbol{e}}$ and covariance matrix $\boldsymbol{\Sigma}$.

Pricing of risky assets

Proposition

If short positions are not allowed, i.e.

$$x^{\ell}_{n,t} \in (0,1) \qquad \forall n \in \mathcal{N},\ \forall \ell \in \mathcal{L},\ \forall t$$

then prevailing prices exist, are unique and strictly positive. It holds:

$$p^{\ell}_t = p^{\ell}_{t-1} \cdot \frac{x^{\ell}_t}{x^{\ell}_{t-1}} \cdot \frac{x^0_{t-1} \cdot (1 + r_f) + \sum_{\ell=1}^{L} x^{\ell}_{t-1} \cdot e^{\ell}_t}{x^0_t}$$

Assumption

No trader can take short position in any asset, i.e. the image of traders' investment functions is restricted such that

$$f_n : \mathbb{R}^{\tau \times L} \longrightarrow \mathrm{Int}\left(\triangle^L\right)$$

Definition

Individual wealth shares:

$$\varphi_{n,t} = \frac{W_{n,t}}{\sum_{n=1}^{N} W_{n,t}}$$

- A trader n is said to *'survive'* the economy if his long-run wealth-share is significantly different from 0, i.e. if $\lim_{t\to\infty} \varphi_{n,t} > 0$
- A trader n is said to *'dominate'* the economy if his long-run wealth-share is significantly close to 1, i.e. if $\lim_{t\to\infty} \varphi_{n,t} = 1$

Following Anufriev et al. (2006), two types of equilibria are possible:

1. Single-survivor equilibria (most '*aggressive*' trader)
2. Multiple-survivor equilibria (non-generic)

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Simulation results - survival patterns

Description	Value
Initial population size	$N = 200$
Number of risky assets	$L = 1$
Static population	`true`
Riskless rate of return	$r_f = 0.02$
γ distribution	$\gamma_n \sim \mathcal{U}(1.0, 1000.0)$
λ distribution	$\lambda_n = 0.1, \ \forall n \in \mathcal{N}$
d distribution	$d_n = 1.0, \ \forall n \in \mathcal{N}$
Initial wealth endowment	$W_{n,0} = 50.0, \ \forall n \in \mathcal{N}$
Yield mean	$\overline{e} = 0.04$
Yield variance	$\sigma_e^2 = 1.0\text{e-}4$
Yield realisation distribution	$e_t \sim \mathcal{N}(\overline{e}, \sigma_e^2)$
Initial risky asset price level	$p_0 = 0.1$
x_n admissible interval	$x_{n,t} \in [0.01, 0.99], \ \forall n \in \mathcal{N}, \forall t$

Table: Parameters and initial conditions (1)

Simulation results - survival patterns (cont'd)

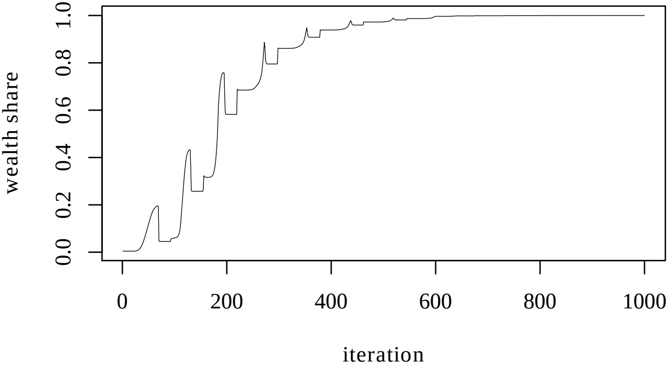


Figure: Evolution of wealth-share for the least-risk-averse trader. Single-survivor.

Description	Value
Initial population size	$N = 200$
Number of risky assets	$L = 1$
Static population	`true`
Riskless rate of return	$r_f = 0.02$
γ distribution	$\boldsymbol{\gamma_n \sim \mathcal{U}(100.0, 1000.0)}$
λ distribution	$\lambda_n = 0.1, \ \forall n \in \mathcal{N}$
d distribution	$d_n = 1.0, \ \forall n \in \mathcal{N}$
Initial wealth endowment	$W_{n,0} = 50.0, \ \forall n \in \mathcal{N}$
Yield mean	$\overline{e} = 0.04$
Yield variance	$\sigma_e^2 = 1.0\text{e-}4$
Yield realisation distribution	$e_t \sim \mathcal{N}(\overline{e}, \sigma_e^2)$
Initial risky asset price level	$p_0 = 0.1$
x_n admissible interval	$x_{n,t} \in [0.01, 0.99], \ \forall n \in \mathcal{N}, \forall t$

Table: Parameters and initial conditions (2)

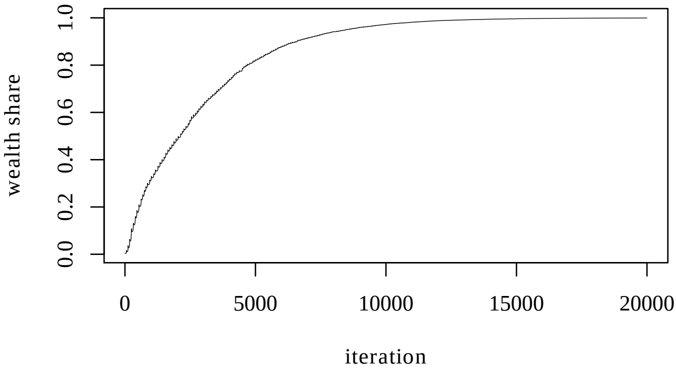
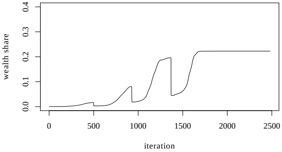


Figure: Evolution of wealth-share for the least-risk-averse trader. Single-survivor.

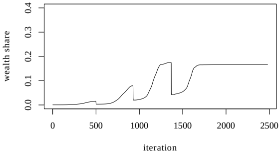
Simulation results - survival patterns (cont'd)

Description	Value
Initial population size	$N = 200$
Number of risky assets	$L = 1$
Static population	`true`
Riskless rate of return	$r_f = 0.02$
γ distribution	$\boldsymbol{\gamma_n \sim \mathcal{U}(1.0, 1000.0)}$
λ distribution	$\boldsymbol{\lambda_n = 0.01,\ \forall n \in \mathcal{N}}$
d distribution	$d_n = 1.0,\ \forall n \in \mathcal{N}$
Initial wealth endowment	$W_{n,0} = 50.0,\ \forall n \in \mathcal{N}$
Yield mean	$\overline{e} = 0.04$
Yield variance	$\sigma_e^2 = 1.0\text{e-}4$
Yield realisation distribution	$e_t \sim \mathcal{N}(\overline{e}, \sigma_e^2)$
Initial risky asset price level	$p_0 = 0.1$
x_n admissible interval	$x_{n,t} \in [0.01, 0.99],\ \forall n \in \mathcal{N}, \forall t$

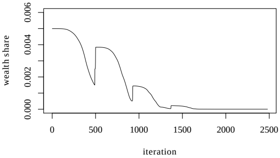
Table: Parameters and initial conditions (3)



(a) lowest γ_n



(b) second-lowest γ_n



(c) highest γ_n

Figure: Multiple-survivor equilibrium. Evolution of wealth-shares.

Simulation results - survival patterns (cont'd)

Heterogeneity in the risk-aversion coefficient, within the stability domain of the system, triggers a wealth-driven selection mechanism.

- For a short memory-span (large λ), the least risk-averse trader survives and dominates the economy;
- For a long memory-span (small λ), multiple traders, still low-risk averse, survive and display identical investment decisions.

Out of the stability domain of the system (i.e. for large enough λ, following Anufriev et al. 2006) selection does not occur: individual wealth-shares keep fluctuating indefinitely with no clear-cut outcome.

Simulation results: price dynamics

Description	Value
Initial population size	$\mathbf{N = 1,000}$
Number of risky assets	$L = 1$
Static population	`true`
Riskless rate of return	$r_f = 0.02$
γ distribution	$\boldsymbol{\gamma_n \sim \mathcal{U}(1.0, 500.0)}$
λ distribution	$\boldsymbol{\lambda_n = 0.0036, \ \forall n \in \mathcal{N}}$
d distribution	$d_n = 1.0, \ \forall n \in \mathcal{N}$
Initial wealth endowment	$W_{n,0} = 50.0, \ \forall n \in \mathcal{N}$
Yield mean	$\overline{e} = 0.04$
Yield variance	$\sigma_e^2 = 1.0\text{e-}4$
Yield realisation distribution	$e_t \sim \mathcal{N}(\overline{e}, \sigma_e^2)$
Initial risky asset price level	$p_0 = 0.1$
x_n admissible interval	$x_{n,t} \in [0.01, 0.99], \ \forall n \in \mathcal{N}, \forall t$

Table: Parameters and initial conditions (4)

Simulation results: price dynamics (cont'd)

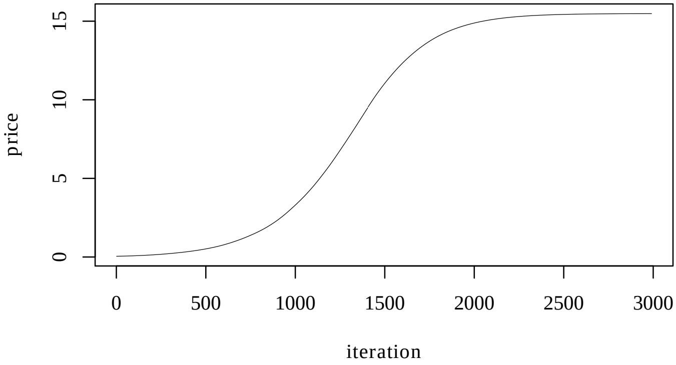


Figure: Price dynamics. Smooth and monotone convergence to equilibrium.

Simulation results: price dynamics (cont'd)

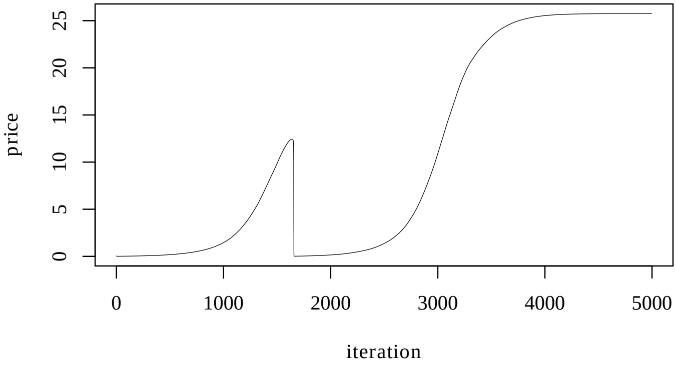


Figure: Price dynamics. Emergence of a bubble-and-bust cycle. $\boldsymbol{\lambda_n = 0.00365}$

Simulation results: price dynamics (cont'd)

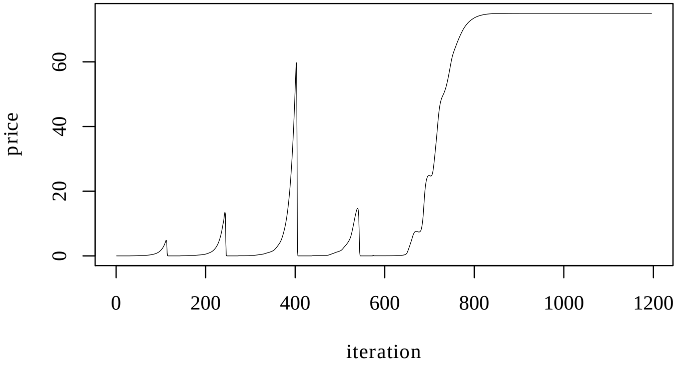


Figure: Price dynamics. Emergence of multiple bubble-and-bust cycles. $\boldsymbol{\lambda_n = 0.155}$

Simulation results: price dynamics (cont'd)

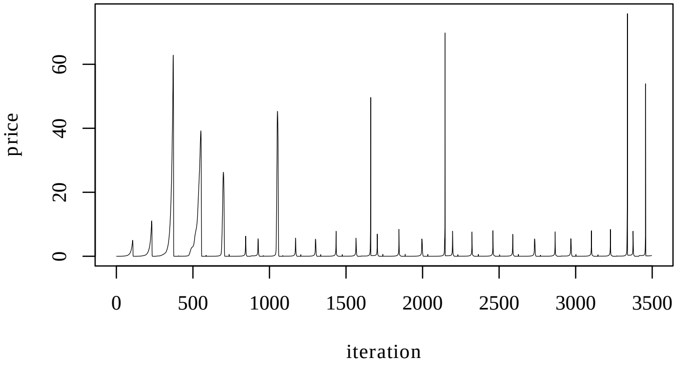


Figure: Price dynamics. No convergence to an equilibrium value. $\lambda_n = 0.16$

Simulation results: price dynamics (cont'd)

The emergent properties observed in a trend-chasers-only setting maintain robustness with respect to the introduction of fundamentalist and trend-contrarian traders in the economy.

- Fundamentalists are expected to stabilise the price as they act against chartists whenever current price deviates from its fundamental value;
- Contrarians shall counteract the attempt made by trend-chasers to exacerbate the price trend by acting in a symmetrical fashion.

We differentiate the d parameter in order to model a population largely composed of quasi-fundamentalists and well balanced crowds of trend-chasers and contrarians.

Simulation results: price dynamics (cont'd)

We now shift the analysis to the transitional price dynamics:

Description	Value
Initial population size	$N = 1,000$
Number of risky assets	$L = 1$
Static population	`true`
Riskless rate of return	$r_f = 0.02$
γ distribution	$\gamma_n \sim \mathcal{U}(1.0, 500.0)$
λ distribution	$\lambda_n = 0.1, \ \forall n \in \mathcal{N}$
d distribution	$\boldsymbol{d_n \sim \mathcal{N}(0, 1)}$
Initial wealth endowment	$W_{n,0} = 50.0, \ \forall n \in \mathcal{N}$
Yield mean	$\bar{e} = 0.04$
Yield variance	$\sigma_e^2 = 1.0\text{e-}4$
Yield realisation distribution	$e_t \sim \mathcal{N}(\bar{e}, \sigma_e^2)$
Initial risky asset price level	$p_0 = 0.1$
x_n admissible interval	$x_{n,t} \in [0.01, 0.99], \ \forall n \in \mathcal{N}, \forall t$

Table: Parameters and initial conditions (5)

Simulation results: price dynamics (cont'd)

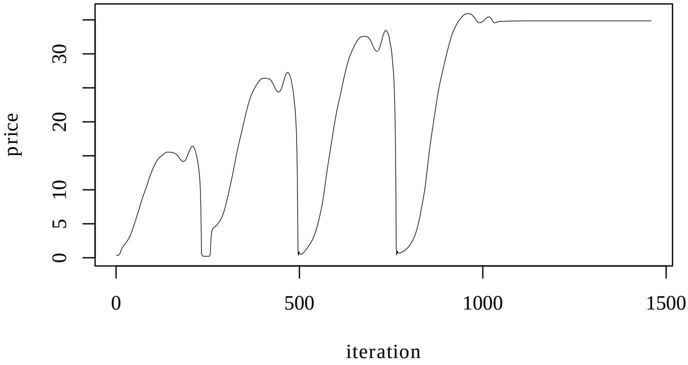


Figure: Price dynamics. Fundamentalists vs. chartists.

Simulation results: price dynamics (cont'd)

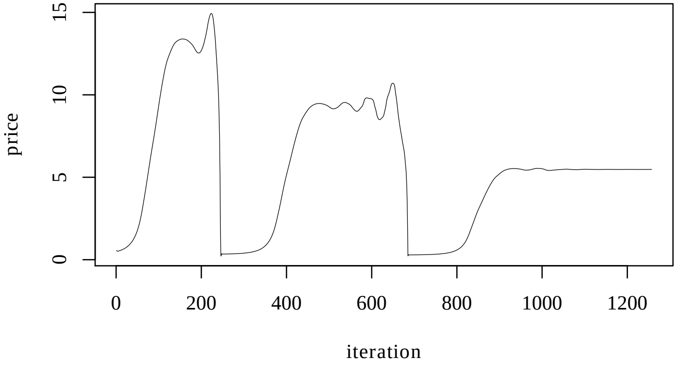


Figure: Price dynamics. Fundamentalists vs. chartists. $\gamma_n \sim \mathcal{U}(1.0, 1000.0)$

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Concluding remarks

1. Analysis of individual wealth-shares
   - strong market selection mechanism
   - single- and multiple-survivor equilibria
   - riskier investment functions globally dominate
2. Analysis of transitional price dynamics
   - emergence of bubble-and-bust cycles
   - robust to the introduction of fundamentalist and trend-contrarian

In a nutshell

risk-aversion heterogeneity

$\Downarrow \quad \Downarrow \quad \Downarrow$

decoupling of price dynamics from the fundamental yield process

Conceivable improvements

Our framework can be extended in a number of directions:

- multiple risky assets
- dynamic population
- more realistic traders' behaviour (prospect theory, herding) and learning (genetic algorithms, classifier systems)

Our conjecture is that a sharper departure from rationality assumption is needed in order to obtain dynamics that are closer to reality.

Selected references

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