

# Wealth-driven asymptotic survival in a financial market with demand shocks

Pietro Dindo<sup>†</sup>

Jacopo Staccioli<sup>‡</sup>

<sup>†</sup> *Department of Economics - University of Venice*

<sup>‡</sup> *Institute of Economics - Scuola Superiore Sant'Anna, Pisa*

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- 1 **Context and motivation**
- 2 The model
- 3 Long-run outcomes
- 4 Simulation
- 5 Concluding remarks

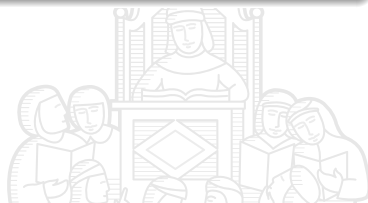


## Friedman 'as if' argument (1953)

- economic agents can be described *as if* they were fully rational
- non-rational agents would get wiped out of the market

## Empirical evidence

- trade occurs
- excess volatility
- market sentiment



## Heterogeneous Agent Models (HAMs)

- analytical investigation of long-run dynamics
- incorporate various degrees of bounded rationality

# Context and motivation

## traditional HAMs

- feedback mechanisms from realised market outcomes
  - e.g. fundamentalist vs. chartist
- long-run analysis within a notion of equilibrium
  - i.e. actions are fixed at the equilibrium
- ‘*deterministic skeleton*’ approach

what happens if one introduces *persistent* demand shocks?

## Black (1986) JoF

noise traders: “*trading on noise as if it were information*”

## Noise traders vs. rational arbitrageurs

De Long et al. (1990) JPE price impact, fixed wealth

De Long et al. (1991) JoB endogenous wealth, no price impact

## Our contribution

- constant vs. stochastic portfolio strategies
- wealth-driven market selection
- conditions for survival and dominance

## Main findings

- trade-off between portfolio riskiness and variability
- long-run heterogeneity
- non-trivial price-wealth dynamics (e.g. volatility clustering)
- loss of generality in adopting the deterministic skeleton approach

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# The model

Consider a stylised financial market (no consumption)

- trade takes place in discrete time  $t$
- risk-free bond
  - perfectly elastic supply
  - constant  $r_f$  return
  - price normalised to 1 (*numéraire*)
- long-lived risky security
  - positive dividend  $d_t = d_{t-1}(1 + g)$ ,  $g > r_f$
  - unitary constant supply
  - market clearing price  $p_t$

At each time step  $t$  trader  $n$

- invests a fraction  $x_{n,t}$  of her wealth  $w_{n,t}$  into the risky security
- residually invests  $(1 - x_{n,t}) \cdot w_{n,t}$  into the bond

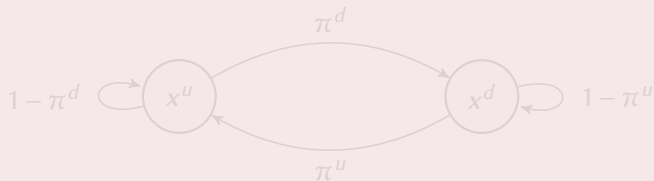
Trader  $n$  wealth  $w_{n,t}$  equals the current market value of her portfolio



# Traders' behaviour

## Assumption

- constant trader always invests  $\bar{x} \in (0, 1)$
- stochastic trader invests according to Markov process  $\{\mathcal{X}_t, t \in \mathbb{N}\}$



$$\pi^u > 0$$

$$\pi^d > 0$$

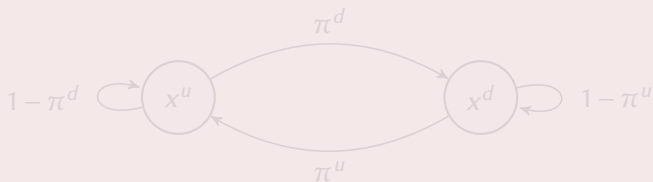
$$0 < x^d < x^u < 1$$



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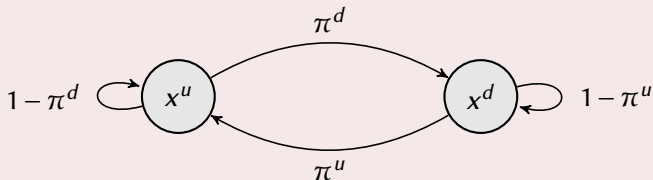
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# Laws of motion of the economy

$$\mathcal{F}_{x_{t-1}, x_t} : \mathcal{D} \rightarrow \mathcal{D}, \quad \mathcal{D} = \Delta \times (-1, +\infty) \times \mathbb{R}_{++}$$

$$\left\{ \begin{array}{l} \varphi_t = \varphi_{t-1} \frac{1 + x_{t-1}(r_t + e_t)}{1 + (r_t + e_t)[\varphi_{t-1}x_{t-1} + (1 - \varphi_{t-1})\bar{x}]} \\ r_t = \frac{\varphi_{t-1}[x_t(1 + e_t x_{t-1}) - x_{t-1}] + (1 - \varphi_{t-1})e_t \bar{x}^2}{\varphi_{t-1}x_{t-1}(1 - x_t) + (1 - \varphi_{t-1})\bar{x}(1 - \bar{x})} \\ e_t = e_{t-1} \frac{1 + g}{1 + r_{t-1}} \end{array} \right.$$

where

$$\varphi_t = \frac{\text{wealth of stochastic trader}}{\text{total wealth}} \in [0, 1]$$

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## Terminology

- trader  $n$  is said to *survive* on  $\{x_t\}_{t=0}^{\infty}$  if  $\limsup_{t \rightarrow \infty} \varphi_{n,t} > 0$
- trader  $n$  is said to *vanish* on  $\{x_t\}_{t=0}^{\infty}$  if  $\limsup_{t \rightarrow \infty} \varphi_{n,t} = 0$
- trader  $n$  is said to *dominate* on  $\{x_t\}_{t=0}^{\infty}$  if  $\liminf_{t \rightarrow \infty} \varphi_{n,t} = 1$

## Proposition

If  $(\varphi^*, r^*, e^*) \in \mathcal{D}$  is a fixed point of system  $\mathcal{F}$  then either

- 1 the constant trader dominates (steady state  $\mathbb{C}$ )
- 2 the stochastic trader dominates (steady state  $\mathbb{S}$ )

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# A dominant constant trader

$$\varphi^c = 0$$

$$r^c = g$$

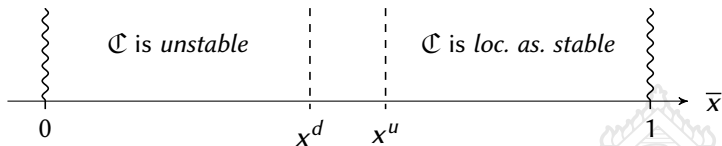
$$e^c = g \frac{1 - \bar{x}}{\bar{x}}$$

## Proposition

Steady state  $\mathcal{C}$  is locally asymptotically stable if

$$\lambda^c = \frac{(\bar{x} + gx^u)^{\frac{\pi^u}{\pi^u + \pi^d}} (\bar{x} + gx^d)^{\frac{\pi^d}{\pi^u + \pi^d}}}{\bar{x}(1 + g)} < 1$$

# A dominant constant trader

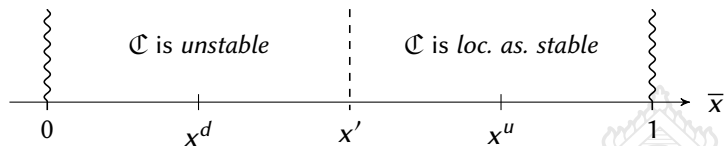


## Sufficient conditions

- $\bar{x} \geq x^u \implies \mathcal{C}$  is locally asymptotically stable
- $\bar{x} \leq x^d \implies \mathcal{C}$  is unstable



# A dominant constant trader



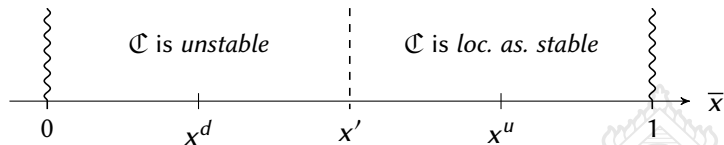
## Proposition

$\exists!$   $x' \in (0, 1)$  such that

- $\forall \bar{x} > x'$ ,  $\mathcal{C}$  is locally asymptotically stable
- $\forall \bar{x} < x'$ ,  $\mathcal{C}$  is unstable

Moreover  $x^d < x' < x^u$

# A dominant constant trader



## Special case

$$\pi^u = \pi^d \implies x' = \frac{x^u + x^d}{2} - h(g)$$

$$h(0) = 0$$

$$h(g) > 0$$

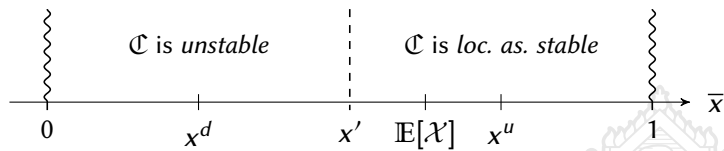
$$h'(g) > 0$$

## Proposition

$x' < \mathbb{E}[\mathcal{X}]$  even if  $\pi^u \neq \pi^d$



# A dominant constant trader



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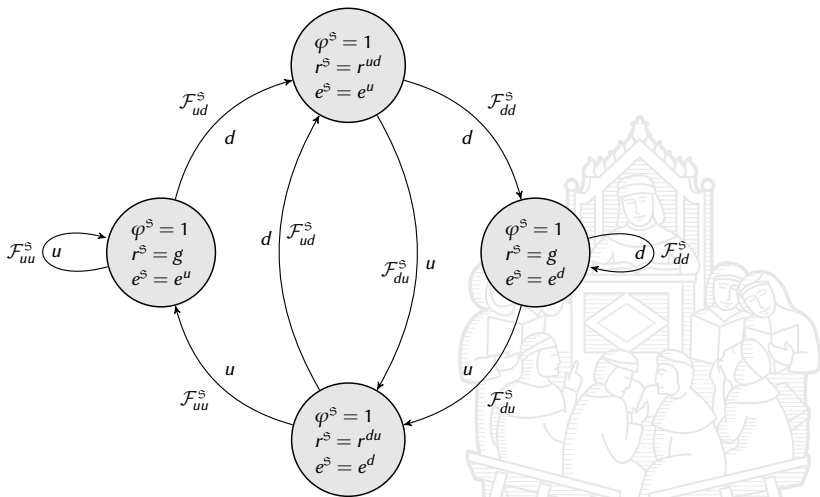
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# A dominant stochastic trader



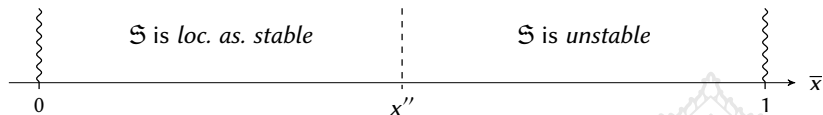
It holds  $r^{ud} < g < r^{du}$  and  $e^u < e^d$ . Moreover, 'usually'  $r^{ud} < 0$

## Proposition

Steady state  $\mathfrak{S}$  is locally asymptotically stable if

$$\lambda^{\mathfrak{S}} = \frac{1}{1+g} \left[ 1 + \frac{g\bar{x}}{x^u} \right]^{\frac{\pi^u(1-\pi^d)}{\pi^u+\pi^d}}$$
$$\cdot \left[ \frac{\bar{x} [g(1-x^u) - (x^u - x^d)] + x^u(1-x^d)}{x^u(1-x^u)} \cdot \frac{\bar{x} [g(1-x^d) + (x^u - x^d)] + x^d(1-x^u)}{x^d(1-x^d)} \right]^{\frac{\pi^u\pi^d}{\pi^u+\pi^d}}$$
$$\cdot \left[ 1 + \frac{g\bar{x}}{x^d} \right]^{\frac{\pi^d(1-\pi^u)}{\pi^u+\pi^d}} < 1$$

# A dominant stochastic trader

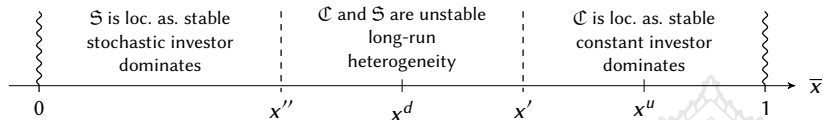


## Numerical result

$\exists! x'' \in (0, 1)$  such that

- $\forall \bar{x} < x''$ ,  $\mathcal{S}$  is locally asymptotically stable
- $\forall \bar{x} > x''$ ,  $\mathcal{S}$  is unstable

# Long-run heterogeneity

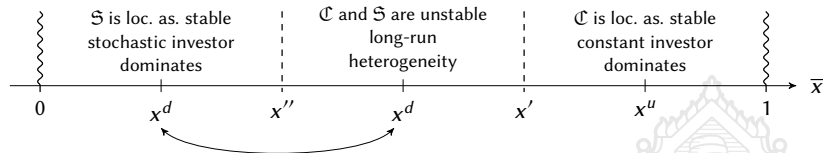


## Proposition

$\exists \hat{g} > 0$  such that  $\forall g < \hat{g}$  it holds  $x'' < x^d < x'$ . In particular

$$\hat{g} = \frac{x^u - x^d}{1 - x^u}$$

# Long-run heterogeneity



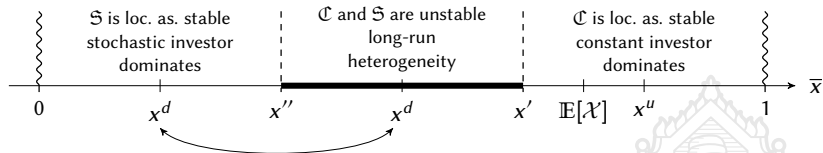
## Numerical result

the relation  $x'' < x'$  holds

- $\forall x^u, x^d, \pi^u, \pi^d \in \{0.01, 0.02, \dots, 0.99\}$  such that  $x^d < x^u$
- $\forall g = \hat{g} \cdot 10^k, k \in \mathbb{N}_+$  such that  $g \leq 10$



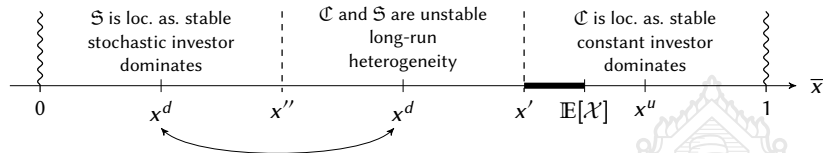
# Long-run heterogeneity



## Corollary

- 1 there exists a **non-degenerate** interval  $(x'', x')$  such that both  $\mathcal{C}$  and  $\mathcal{S}$  are unstable  $\forall \bar{x} \in (x'', x')$
- 2 the constant trader is able to *dominate* the stochastic trader even adopting a **on-average safer** position  $\bar{x} < \mathbb{E}[\mathcal{X}]$
- 3 the constant trader is able to *invade* the stochastic trader even adopting an **always strictly safer** position  $\bar{x} < x^d < x^u$  when  $x'' < x^d$

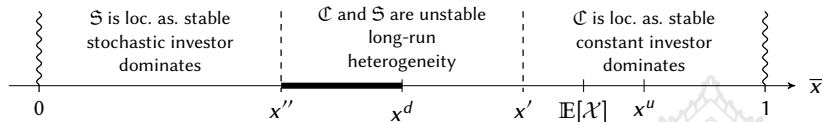
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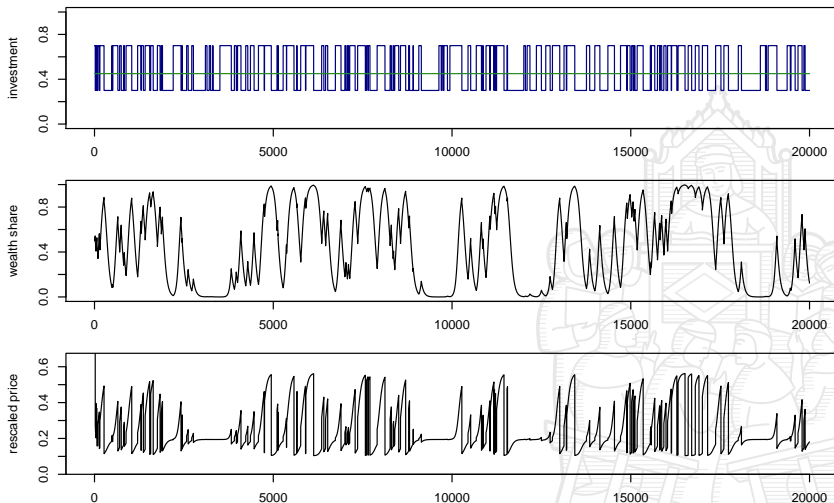
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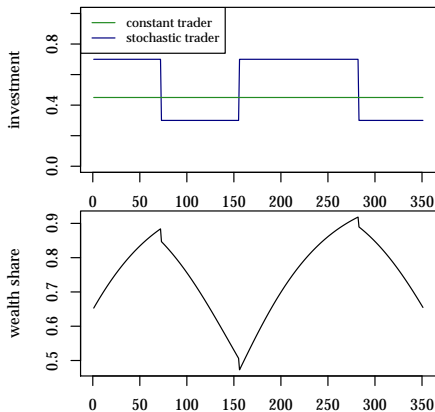


Description	Variable	Value
stochastic investment up	$x^u$	0.7
stochastic investment down	$x^d$	0.3
probability down when up	$\pi^d$	0.01
probability up when down	$\pi^u$	0.01
constant investment	$\bar{x}$	0.45
dividend rate of growth	$g$	0.05
initial wealth share	$\varphi_0$	0.5
initial return	$r_0$	0.0
initial yield	$e_0$	0.01

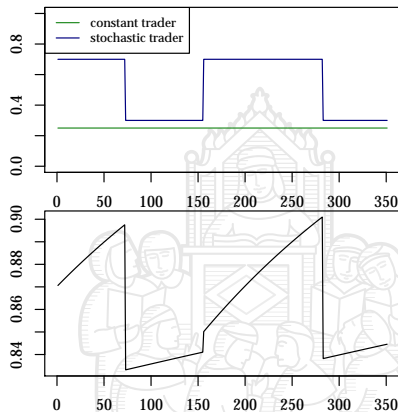
# Simulation



# A tale of two regimes

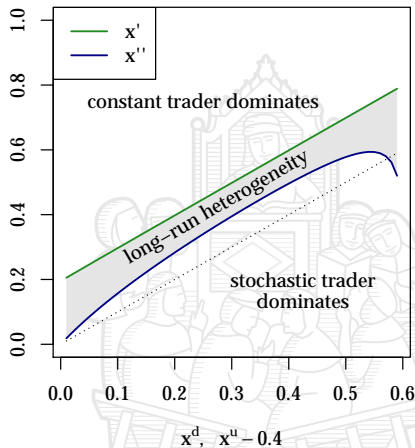
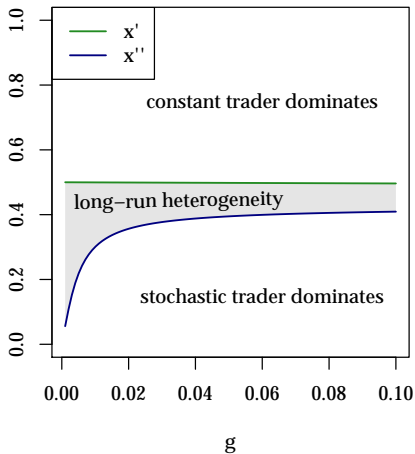


1st regime:  $g = 0.05$  and  $\bar{x} = 0.45$



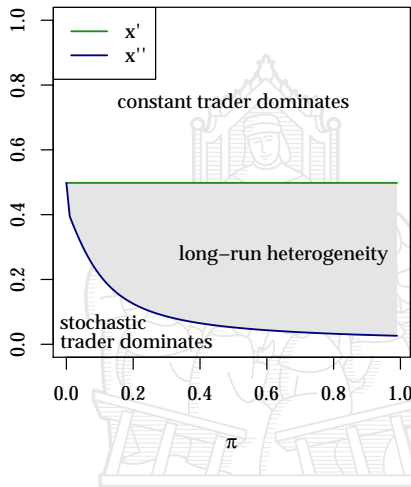
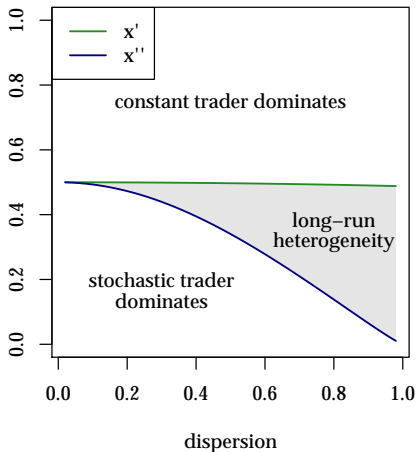
2nd regime:  $g = 0.005$  and  $\bar{x} = 0.25$

# Sensitivity analysis





# Sensitivity analysis

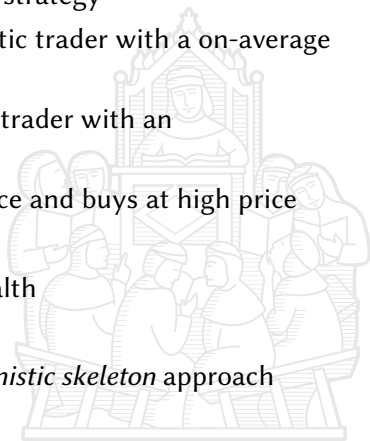


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# Concluding remarks

- trade-off between portfolio riskiness and variability
- $\exists$  intrinsic penalty in adopting stochastic strategy
- a constant trader can dominate a stochastic trader with a on-average safer portfolio
- a constant trader can invade a stochastic trader with an always-strictly-safer portfolio
- the stochastic trader (fire-)sells at low price and buys at high price
- generic long-run heterogeneity
- endogenous fluctuations of price and wealth
- volatility clustering
- loss of generality in adopting the *deterministic skeleton* approach



Hope you slept comfortably

Thank you very much!

[j.staccioli@sssup.it](mailto:j.staccioli@sssup.it)

...and props to the European Project PEOF-GA-2011-300637 MSAEO for financial support