

Economics of Information

LECTURE 6

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competitive equilibrium

a situation in which prices are such that all arbitrage profits are eliminated

- is it possible for a competitive economy to always be in equilibrium?
- no, because arbitrageurs make no return from their costly activity

“the assumptions that all markets, including that for information, are always in equilibrium and always perfectly arbitrated are inconsistent when arbitrage is costly”

[ibidem, p. 393]

Grossman and Stiglitz (1980)

- market with an '*equilibrium amount of disequilibrium*'
- price reflects information only *partially*
- those who acquire costly information do receive compensation
- those who don't, partially receive information anyway
- informativeness of the price system depends on # informed
- # informed individuals is endogenous
- if prices fully reflect information (cf. EMH) there is no equilibrium

The model

- two assets are traded in a market
 - a *safe* asset, yielding constant return R
 - a *risky* asset, yielding return $u = \theta + \varepsilon$
- both θ and ε are random variables
- θ is observable at a cost $c > 0$ while ε is unobservable
- *ex ante* identical individuals choose to be informed or uninformed
- individuals have rational expectations
- demand of informed traders depends on θ and price P
- demand of uninformed traders depends only on P
- random variable x is the supply of risky asset
- uninformed traders cannot observe x

equilibrium price

- define λ as the percentage of informed traders
- equilibrium is $P_\lambda(\theta, x)$ such that demand equals supply
- uninformed are prevented from learning θ by observing $P_\lambda(\theta, x)$
- they cannot distinguish between variations in P due to θ or x
- depending on relative variances of the RVs, price reveals ‘something’

Conjectures

- 1 the higher λ , the more informative the price system (in terms of θ)
- 2 the higher λ , the lower EU_I/EU_U
- 3 the higher c , the lower λ
- 4 the higher the quality of information available to informed traders ($\sigma_\theta^2 \uparrow$ or $\sigma_\varepsilon^2 \downarrow$), the more informative the price system
- 5 the higher the noise ($\sigma_x^2 \uparrow$), the less informative the price system
- 6 in the limit $\sigma_x^2 \rightarrow 0$ a competitive equilibrium does *not* exist
- 7 markets are thinner when λ is close to the extremes of Δ

Price cannot fully reflect costly information

suppose c is small enough so that it is worthwhile for a trader to become informed when no other trader is informed. If there is no noise ($\sigma_x^2 = 0$) then there doesn't exist an equilibrium

- if $\sigma_x^2 \downarrow$ then informativeness of price system grows ($\text{Var}[u|w_\lambda] \downarrow$) and for the equilibrium to be maintained it must be that $\lambda \downarrow$;
- in the limit for $\sigma_x^2 \rightarrow 0$, all the information purchased by informed traders is transmitted to uninformed traders by the price system
- each informed trader, acting as a price taker, finds optimal to become uninformed
- $\lambda > 0$ is not an equilibrium
- but if nobody is informed, no trader learns from the price system
- if c is low, uninformed traders have an incentive to become informed
- $\lambda = 0$ is not an eq.; \exists discontinuity in $\frac{EU_I}{EU_U}$ between $\lambda = 0$ and $\lambda > 0$

Price cannot fully reflect costly information (cont'd)

if information is perfect ($\sigma_\varepsilon^2 = 0$), then there doesn't exist an equilibrium

- as $\sigma_\varepsilon^2 \rightarrow 0$ informed traders' demands become very sensitive to θ
- then also market clearing price becomes very sensitive to θ
- all traders individually desire to be uninformed
- but when everyone is uninformed, it pays off to expend c and become informed

On the thinness of speculative markets

■ why does trade take place?

- differences in beliefs, endowments, preferences
- Grossman (1975, 1977, 1978): preferences are not a major factor
- all traders have the same coefficient of absolute risk aversion a
- assume all traders have the same endowments

■ how much trade takes place?

- beliefs tend to be homogeneous whenever
 - $\lambda \rightarrow 0$ (all traders are uninformed)
 - $\lambda \rightarrow 1$ (all traders are informed)

On the thinness of speculative markets (cont'd)

Theorem

- for sufficiently large or small c , the mean and variance of trade is zero^a
- as the precision of informed traders' information goes to infinity, the mean and variance of trade go to zero

^athe amount of trade is a Normally distributed RV that depends on θ and x

“the result that competitive equilibrium is incompatible with informationally efficient markets should be interpreted as meaning that speculative markets where prices reveal a lot of information will be very thin because it will be composed of individuals with very similar beliefs” [ibidem, p. 403]

On the possibility of perfect markets

- traders have an incentive to become *informed* only if they can use their information to take ‘better’ positions w.r.t. uninformed traders
- the *Efficient Market Hypothesis* (Fama, 1970) asserts that “*at any time prices fully reflect all available information*”
- if this were true, informed traders couldn’t earn a return on their information
- the price system reveals the signal $w_\lambda \equiv \theta - \frac{a\sigma_\varepsilon^2}{\lambda}(x - E[x])$
- when EMH is true (e.g. $\sigma_\varepsilon^2 = 0$ or $\sigma_x^2 = 0$) each informed trader, acting as a price taker, finds optimal to become uninformed
- but when all traders are uninformed, each of them has an incentive to become informed
- no equilibrium exists

On the possibility of perfect markets (cont'd)

Fama (1970)

$c = 0$ is a *sufficient* condition for prices to reveal all available information

Grossman and Stiglitz (1980)

$c = 0$ is a *necessary* condition for prices to reveal all available information

- but this is a *reductiō ad absurdum* !
- competitive markets are important *only* when information is costly !

On the possibility of perfect markets (cont'd)

extension – 2 risky assets

- introduce security z , with price q , that pays

$$z = \begin{cases} 1 & \text{if } u > E[\theta] \\ 0 & \text{if } u \leq E[\theta] \end{cases}$$

- equilibrium condition

$$\lambda X_I(p, q, \theta) + (1 - \lambda) X_U(p, q) = 0$$

- under weak assumptions p and q fully reveal θ
- market would be *noiseless* and no equilibrium would exist

On the possibility of perfect markets (cont'd)

- there *cannot* be complete markets
- if there were, competitive equilibrium would not exist
- it is only because of costly transactions and incomplete markets that competitive equilibrium can be established
- there is a fundamental conflict between the efficiency with which markets spread information and incentives to acquire information

“as soon as the assumptions of the conventional perfect capital markets model are modified to allow even a slight amount of information imperfection and a slight cost of information, the traditional theory becomes untenable”

[*ibidem*, p. 403]

Thank you for your attention!

see you at the exam!

