Economics of Information

Lecture 3

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Preliminary definitions

information set

a player's information set at any particular point of the game is the set of different nodes in the game tree that she knows might be the actual node, but between which she cannot distinguish by direct observation

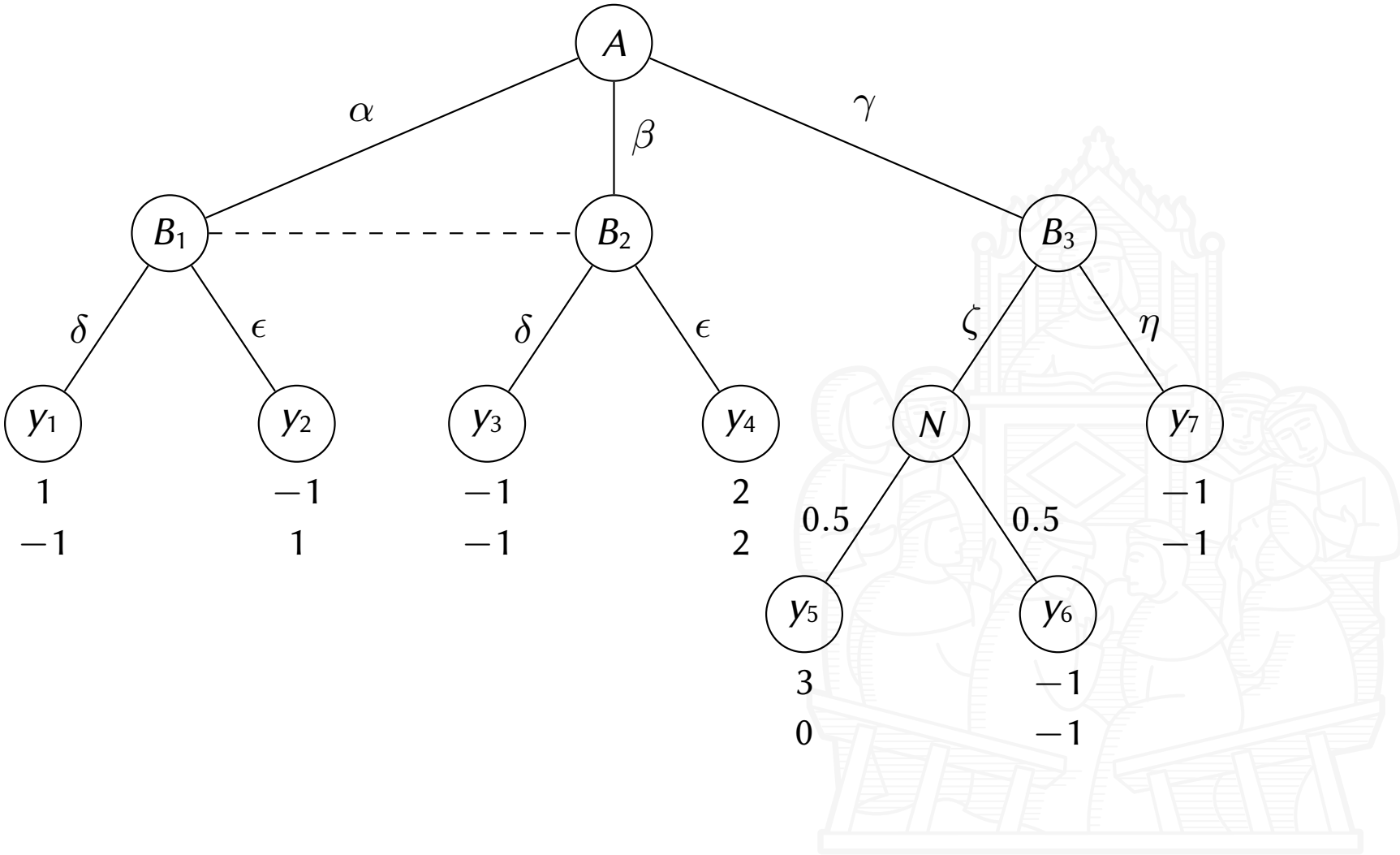
common knowledge

information is common knowledge if it is known to all the players, if each player knows that all the players know it, if each player knows that all the players know that all the players know it, and so forth ad infinitum

Nature

'Nature' is a pseudo-player who takes random actions at specified points in the game with specified probabilities

Example



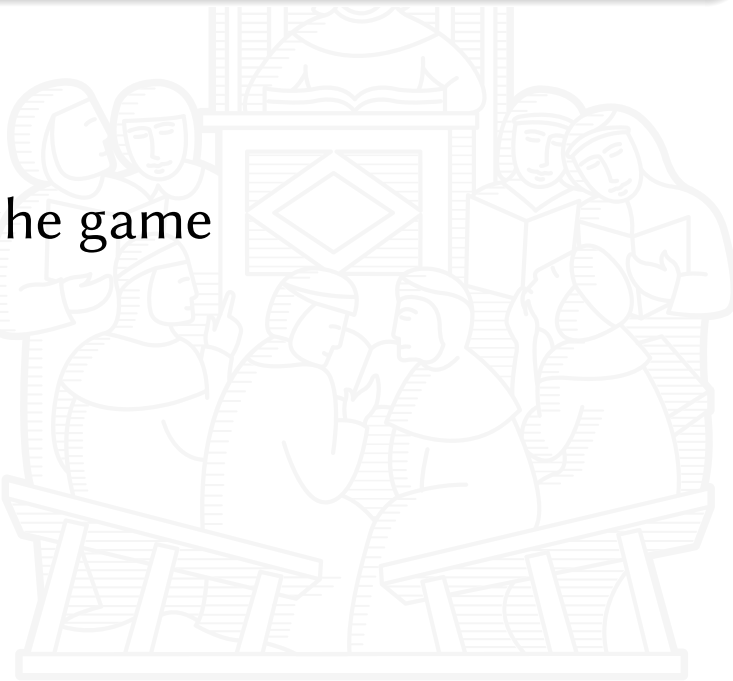
[Im-]Perfect information

Definition

in a game of ***perfect*** *information each information set is a singleton; otherwise the game is one of* ***imperfect*** *information*

perfect information

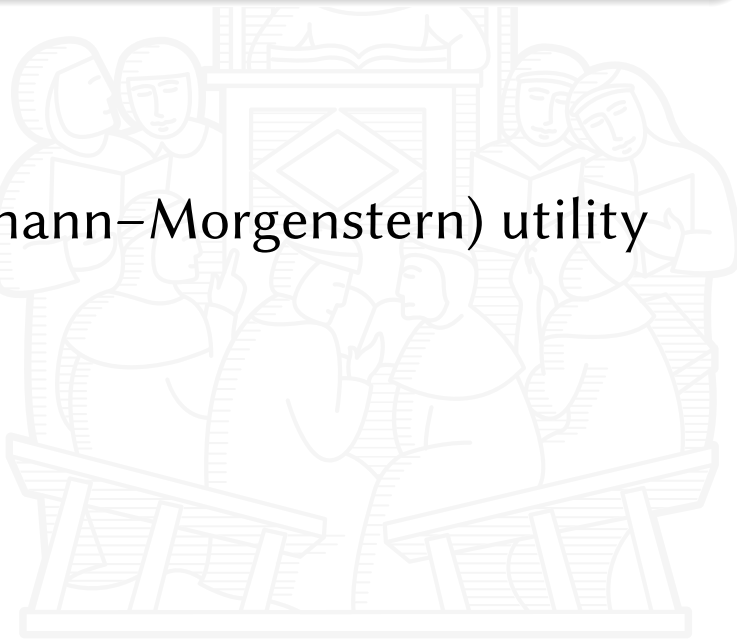
- each player knows exactly her location in the game
- no moves are simultaneous
- all players observe Nature's move, if any



[Un-]Certainty

Definition

a game of ***certainty*** *has no moves by Nature after any player moves; otherwise the game is one of* ***uncertainty***



game of uncertainty

- players maximise their *expected* (von Neumann–Morgenstern) utility
- attitude towards risk matters

[A-]Symmetric information

Definition

in a game of ***symmetric*** *information, a player's information set at*

1. *any node where she chooses an action, or*
2. *an end node*

contains at least the same elements as the information sets of every other player; otherwise the game is one of ***asymmetric*** *information*

asymmetric information

- information is *not* perfect since information sets which differ across players cannot be singletons
- can have moves by Nature but no player has an informational advantage

symmetric information

- can have perfect information if there are no simultaneous moves

[In-]Complete information

Definition

in a game of ***incomplete*** *information, Nature moves first and is unobserved by at least one of the players; otherwise the game is one of* ***complete*** *information*

- a game with incomplete information also has imperfect information, because some player's information set includes more than one node
- two kinds of games have complete but imperfect information
  1. games with simultaneous moves
  2. games where, late in the game, Nature makes moves not immediately revealed to all players

Recap

class	definition
perfect	each information set is a singleton
certain	Nature does not move after any player moves
symmetric	no player has information different from other players when she moves, or at the end nodes
complete	Nature does not move first, or its initial move is observed by every player

Source: Rasmusen (2007, p. 49, tab. 2.4)

Example

Poker game

the players make bets on who will have the best hand of cards at the end, where a ranking of hands has been pre-established

- all cards are dealt face up **PERFECT, CERTAIN, SYMMETRIC, COMPLETE**
- all cards are dealt face down, and a player cannot look even at her own cards before she bets **IMPERFECT, CERTAIN, SYMMETRIC, INCOMPLETE**
- all cards are dealt face down, and a player can look at her own cards **IMPERFECT, CERTAIN, ASYMMETRIC, INCOMPLETE**
- all cards are dealt face up, but each player then scoops up her hand and secretly discards one card **IMPERFECT, CERTAIN, ASYMMETRIC, COMPLETE**
- all cards are dealt face up, the players bet, and then each player receives one more card face up **PERFECT, UNCERTAIN, SYMMETRIC, COMPLETE**
- all cards are dealt face down, but then each player scoops up her cards without looking at them and holds them against her forehead so all the other players can see them (Indian poker) **IMPERFECT, CERTAIN, ASYMMETRIC, INCOMPLETE**

Principal-Agent model

the *principal* hires the *agent* to perform a task, and the agent acquires an informational advantage about her type, her actions, or the outside world at some point in the game

principal is the player with coarser information

agent is the player with finer information

usually it is assumed that the players make a binding *contract* at some point, e.g. the principal commits to paying the agent an agreed sum if she observes a certain outcome

Moral hazard

"any situation in which one person makes the decision about how much risk to take, while someone else bears the cost if things go badly" [Krugman (2009)]

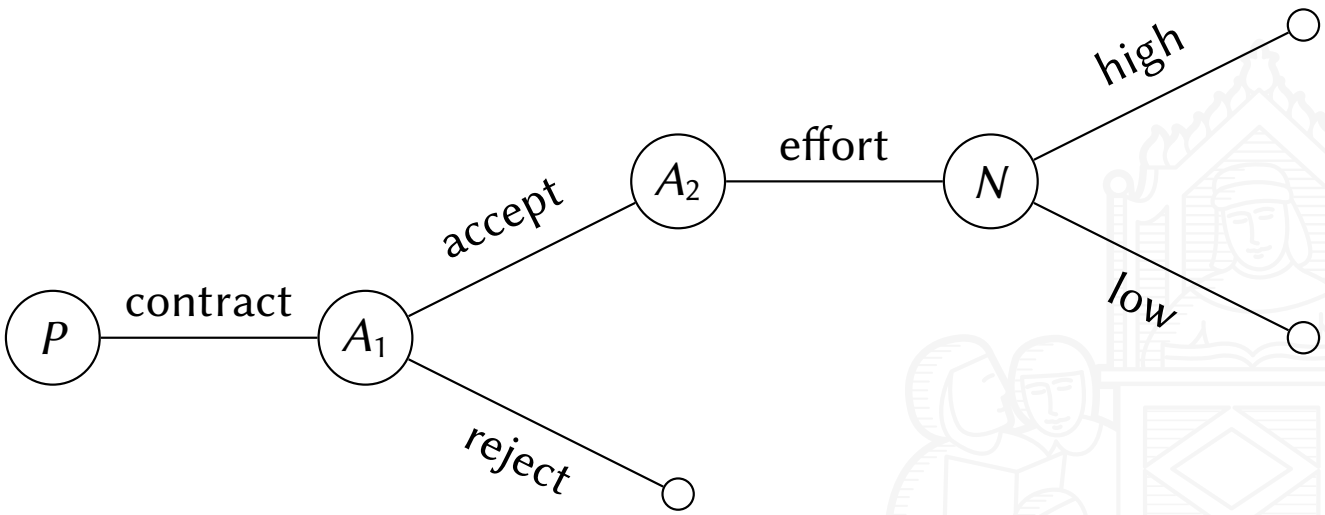
Example

a person with insurance against automobile theft may be less cautious about locking their car since the negative consequences of vehicle theft are now (partially) a liability to the insurance company

1. the principal offers a contract
2. the agent accepts or rejects
3. Nature adds noise to the task being performed

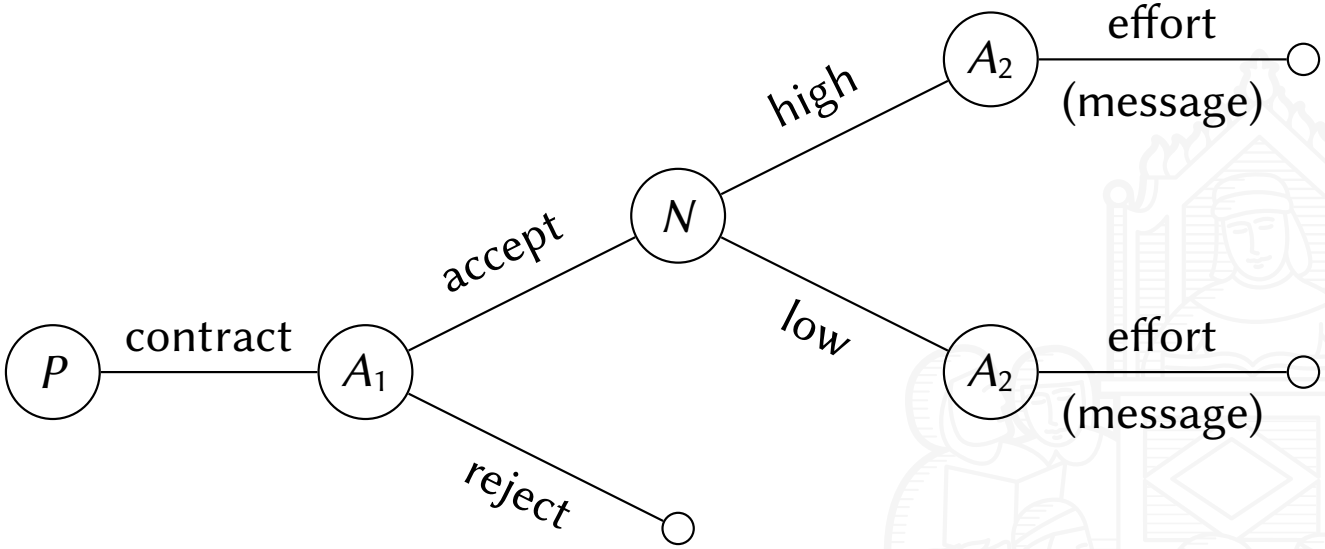
- game of **complete** information with **uncertainty**

Moral hazard with hidden action



Source: Rasmusen (2007, p. 183, fig. 7.1a)

Postcontractual hidden knowledge



Source: Rasmusen (2007, p. 183, fig. 7.1b)

Adverse selection

the agent has private information already **before** the contract is conceived

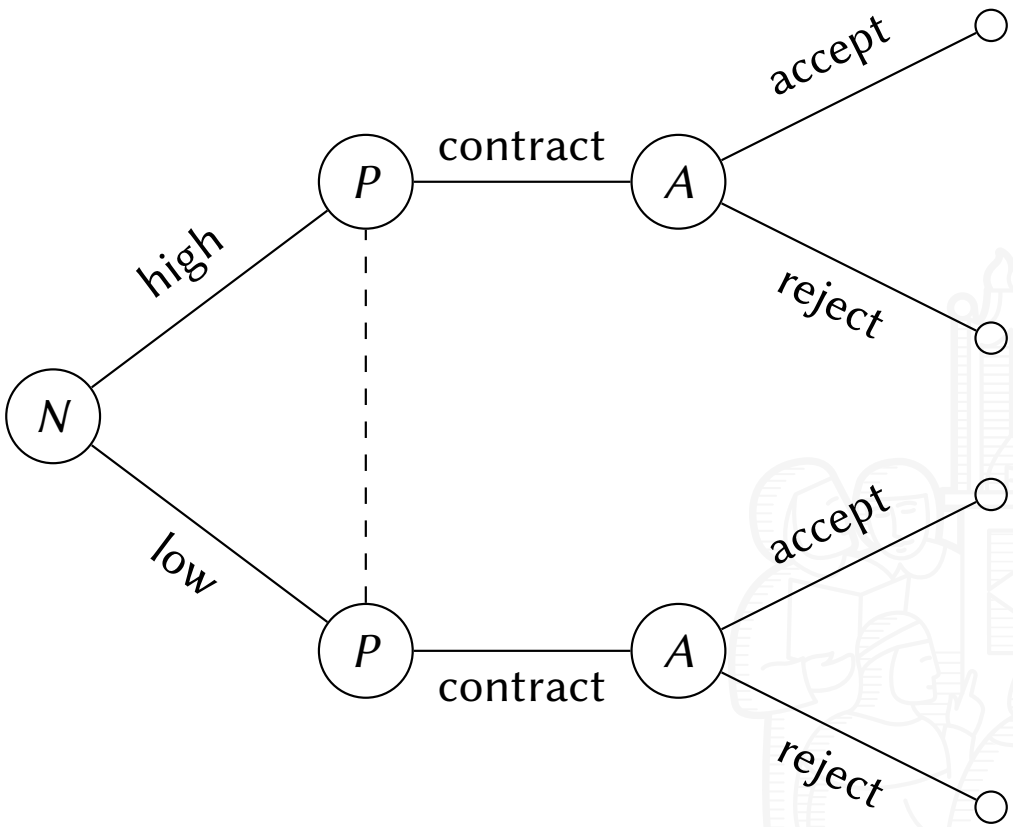
Example

an individual's demand for insurance is positively correlated with the individual's risk of loss

1. Nature picks the agent type
2. the principal offers a contract
3. the agent accepts or rejects

- game of **incomplete** information with **(un-)certainty**

Adverse selection (cont'd)



Source: Rasmusen (2007, p. 183, fig. 7.1c)

Recap

Example

an employer (the principal) hires a worker (the agent)

- if the employer knows the worker's ability but not his effort level: moral hazard with hidden actions
- if neither player knows the worker's ability at first, but the worker discovers it once he start working: postcontractual hidden knowledge
- if the worker knows her ability from the start but the employer doesn't: adverse selection

- these categories are not uniformly recognised
- Myerson (1991) argues that postcontractual hidden knowledge (AKA *'moral hazard with hidden knowledge'*, AKA *'postcontractual adverse selection'*) is essentially the same as adverse selection

The Production Game (PG)

players

principal: a manager

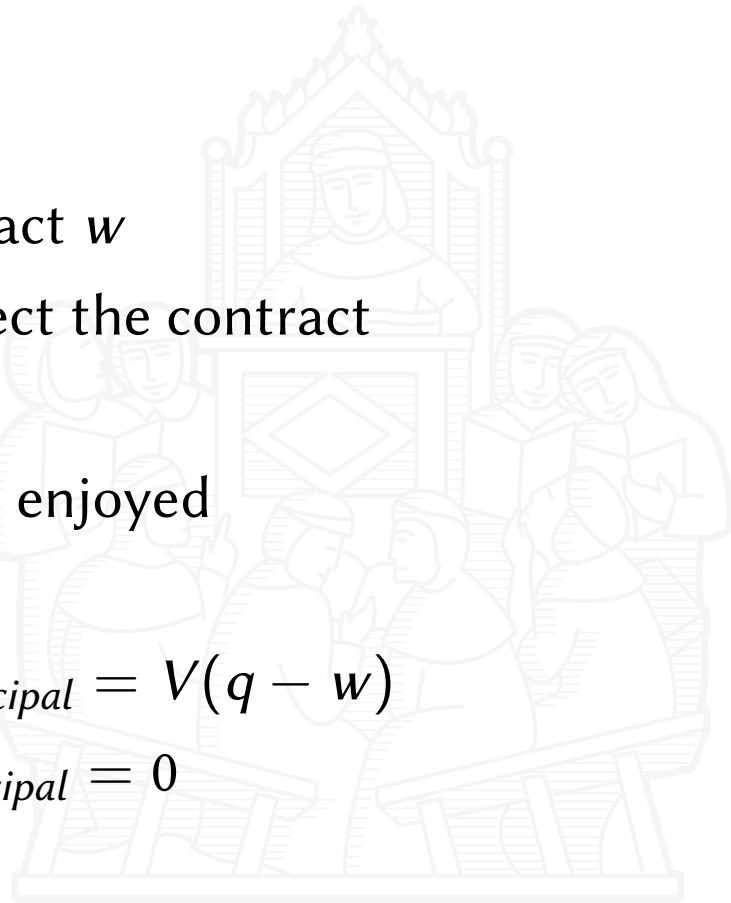
agent: a worker

order of play

1. the principal offers the agent a wage contract w
2. the agent decides whether to accept or reject the contract
3. if the agent accepts, she exerts effort e
4. output q is produced and utility/profits are enjoyed

payoffs

- if agent accepts: $\pi_{agent} = U(e, w)$ $\quad$ $\pi_{principal} = V(q - w)$
- if agent rejects: $\pi_{agent} = \overline{U}$ $\quad$ $\pi_{principal} = 0$



PG III: flat wage under certainty

- the principal observes neither effort nor output
- asymmetric information
- she cannot condition the contract on either e or q
- any conditional contract is unenforceable
  - e.g. a court cannot enforce a contract in which a client agrees to pay a barber \$50 if the haircut is 'especially good' and \$10 otherwise
  - a court can only enforce contingencies it can observe
- only solution is an unconditional flat wage
- if wage is nonnegative the agent accepts but exerts zero effort
- in equilibrium the principal offers zero wage
- the agency problem cannot be solved: moral hazard
- possible solution under reputation or repeated games

PG IV: output-based wage under certainty

- the principal cannot observe effort but observes output
- contract is a function $w(q)$
- since production function is one-to-one the principal computes the optimal effort e^* (as in PG I) and designs the contract using $q^* = q(e^*)$
- e.g. forcing contract, threshold contract… s.t. $U\left(e^*, w(q(e^*))\right) = \overline{U}$
- asymmetric information about effort is not a problem
- the contract is enforceable
- it can be conditioned on something observable and perfectly correlated with effort

PG V: output-based wage under uncertainty

- the principal cannot observe effort but observes output
- output is a function $q(e, \theta),\ \theta \in \mathbb{R}$
- state of the world $\theta \sim f(\theta)$ is chosen by Nature after all other moves
- $q(\cdot)$ is not one-to-one function of effort
- forcing the agent to adopt e^* found under certainty causes the principal to suffer a loss whenever $q < q^*$
- simple contracts (e.g. forcing, threshold) are not efficient
  - low wage when $q \neq q^*$ or $q < q^*$
  - high wage when $q = q^*$ or $q \geq q^*$
- agent participation requires $\mathrm{E}\left[U\left(e, w(q(e))\right)\right] = \overline{U}$
- if agent is risk-averse $\mathrm{E}[w] > w^*$ found under certainty
- trade-off between incentives and insurance against risk
- no contract can induce the agent to exert efficient effort e^* without inducing extra costs (e.g. extra risk on the agent)

PG V: optimal contracts

first-best

a first-best contract achieves the same allocation as the contract that is optimal when the principal and the agent have the same information set and all variables are contractible

second-best

a second-best contract is Pareto optimal given information asymmetry and constraints on writing contracts

- no general answer for finding a second-best contract
- strategy space is 'tricky'
- e.g. principal maximises profit over all possible linear contracts
- but the best contract needs not be linear

PG V: relevant constraints

the principal problem

maximise her own utility knowing that

- the agent is free to reject the contract entirely
- the contract should incentivise the agent to choose the desired effort

$$\max_{w(\cdot)} \; \mathsf{E}\left[V\Big(q(\tilde{e},\theta) - w(q(\tilde{e},\theta))\Big)\right]$$

subject to

$$\mathsf{E}\left[U\Big(\tilde{e}, w(q(\tilde{e},\theta))\Big)\right] \geq \overline{U} \qquad \texttt{(participation)}$$

$$\tilde{e} = \arg\max_{e} \; \mathsf{E}\left[U\Big(e, w(q(e,\theta))\Big)\right] \qquad \texttt{(incentive compatibility)}$$

The relevant constraints (cont'd)

Grossman and Hart (1983): 3-step procedure

1. $\forall e$ find the set of wage contracts that induce the agent to choose e
2. find the contract supporting e that minimises cost to the principal
3. choose e^* that maximises profit, given contracts found in 2

step 1 and 2 can be combined

$$C(\tilde{e}) = \min_{w(\cdot)} \; \mathrm{E}\left[w(q(\tilde{e}, \theta))\right]$$

$$\text{subject to } (\texttt{incentive compatibility}) \text{ and } (\texttt{participation})$$

step 3

$$\max_{\tilde{e}} \; \mathrm{E}\left[V(q(\tilde{e}, \theta) - C(\tilde{e}))\right]$$

Thank you for your attention!

see you on

Monday, 25th March

h. 17:00 – Aula 2 Toscanelli