

# An agent-based model of intra-day financial markets dynamics

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- 1 **Context and motivation**
- 2 Stylised facts
- 3 The model
- 4 Simulations
- 5 Concluding remarks



during recent years, the availability of advanced technology has been substantially reducing the latency required to operate on financial markets, fostering market activity at increasingly higher frequencies

**Cont (2011)** time to execution dropped 25-fold between 2000 and 2010

**Carrion (2013)** 68.3% of NASDAQ dollar turnover attributable to HFT

**Aldridge (2013)** the majority of HFTs delivered positive returns in 2008, whereas 70% of LFTs lost money

## high-frequency traders

- high # of trades per day
- low average gain per trade
- low overnight inventories

## pros? cons?

- market quality
- volatility
- flash-crash

# Stylised facts

## low- or cross-frequency

- properties of returns
- properties of volumes

## high-frequency

- properties of timing and order-flow

## agent-based models

- difficulty in mapping simulation time into calendar time
- no model has yet addressed the high-frequency set of stylised facts

some of the stylised facts have been already (singularly) investigated and linked to patterns of information diffusion

## our proposal

- parsimonious financial agent-based model
- intra-day financial dynamics
- no role for information diffusion
- most of the stylised facts *jointly* emerge from the endogenous interaction of heterogeneous traders

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**SF1 LEPTOKURTOSIS** unconditional distribution displays heavier tail w.r.t. Gaussian distribution

**SF2 NO LINEAR AUTOCORRELATION** positive autocorr. quickly fading away

**SF3 VOLATILITY CLUSTERING** positive autocorr. of absolute/squared value slowly fading away

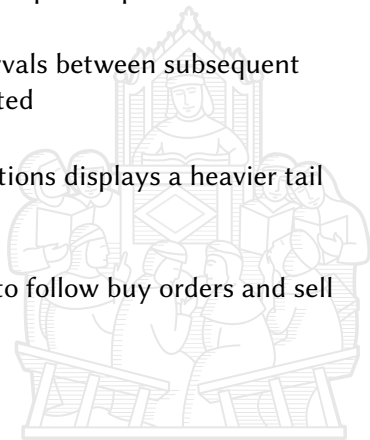
**SF4 LEVERAGE EFFECT** volatility is higher during price drops than during price surges  $\implies$  negative correlation between volatility (absolute returns) and returns

SF5 # PRICE CHANGES PER DAY 10,000+ for blue-chips in liquid markets

SF6 AUTOCORRELATION OF DURATIONS time intervals between subsequent trades are positively autocorrelated

SF7 FAT-TAILED DURATIONS distribution of durations displays a heavier tail w.r.t. exponential distribution

SF8 ORDER-FLOW CLUSTERING buy orders tend to follow buy orders and sell orders tend to follow sell orders

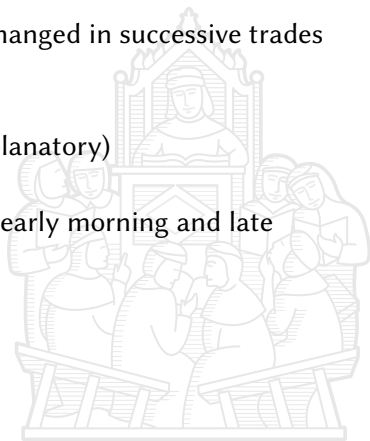




**SF9 VOLUMES AUTOCORRELATION** quantities exchanged in successive trades exhibit positive autocorrelation

**SF10 VOLUME/VOLATILITY CORRELATION** (self-explanatory)

**SF11 U-SHAPED ACTIVITY** volumes peak during early morning and late afternoon



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# Ingredients

- order-driven financial market
    - price-time priority
  - single long-lived security
  - no dividend
  - no fundamental news
  - $N$  heterogeneous agents
    - **fundamentalists**    mean-reverters
    - **chartists**        trend-followers and contrarians
  - no strategy switching
- strict global schedule  $\implies$  Euronext
- endogenous participation based on past volatility
- automatic order cancellation



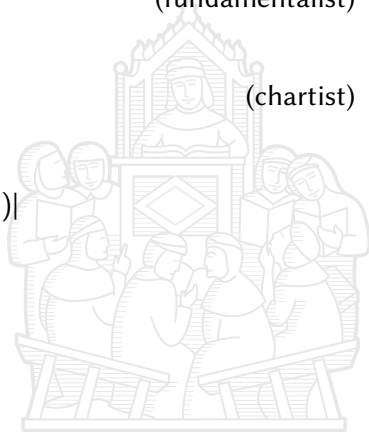
$$\hat{r}_{i,t+h}^F = w_i^F \cdot \log\left(\frac{p^F}{p_t}\right) + \varepsilon_t$$

(fundamentalist)

$$\hat{r}_{i,t+h}^C = w_i^C \cdot \log\left(\frac{p_t}{p_{t-h}}\right) + \varepsilon_t$$

(chartist)

- fundamentalist sensitivity  $w_i^F \sim |\mathcal{N}(0, \sigma_F^2)|$
- chartist sensitivity  $w_i^C \sim \mathcal{N}(\mu_C, \sigma_C^2)$
- fundamental price  $p^F > 0$
- memory/horizon  $h \in \mathbb{N}_+$
- common *i.i.d.* noise  $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2)$



## definition

a limit order submitted by trader  $i$  at time  $t$  is a triple

$$\begin{aligned}\ell_{i,t} &= \{ \text{price, quantity, validity} \} \\ &= \left\{ \text{round}(p_t \cdot \exp(\hat{r}_{i,t+h}), \text{tick}), \text{sgn}(\hat{r}_{i,t+h}), t+h \right\}\end{aligned}$$

- $\text{round}(\cdot)$  is the rounding function
- $\text{tick}$  is the minimum price increment/decrement
- $\text{sgn}(\cdot)$  is the sign function

- no feedback from current time of the day

## automatic cancellation

a stored order  $\ell_{i,t}$  is automatically deleted from the book

- at its expiration time  $t + h$
- if  $i$  submits a new order with different sign (side)
- if  $i$  submits a new order and  $\ell_{i,t}$  is deemed unfavourable
  - new buy order at lower price
  - new sell order at higher price



# Trader participation

## uniform activation

- exactly one trader is activated at each time step, randomly selected from the population  $N$

## endogenous activation

- trader  $i$  is active at time  $t$  if

$$|r_\tau| > \delta_{i,t} \sim |\mathcal{N}(0, \sigma_\delta^2)|$$

where  $\tau < t$  denotes the last time a trade occurred

- if  $|r_\tau| < \delta_{i,t}, \forall i = 1, \dots, N$  then uniform activation with probability  $\phi > 0$

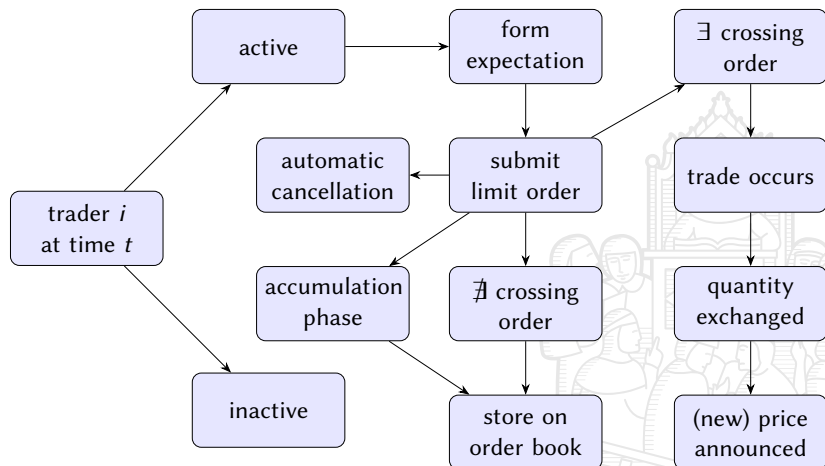
# Timing (EURONEXT)

<b>time</b>	<b>phase</b>	<b>duration</b>
from 7:15am to 9:00am	pre-opening	6,300 s
at 9:00am	opening auction	—
from 9:00am to 5:30pm	main trading session	30,600 s
from 5:30pm to 5:35pm	pre-closing	300 s
at 5:35pm	closing auction	—
10 hours, 20 minutes	—	37,200 s

**1 simulation step**  $\iff$  **1 calendar second**



# Workflow



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# Preliminaries

we simulate the model numerically under three scenarios

**NT** : only noise traders

**FC** : fundamentalists and chartists with uniform participation

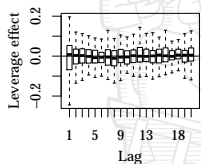
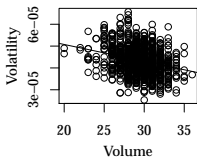
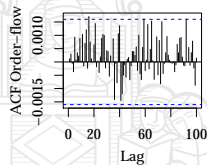
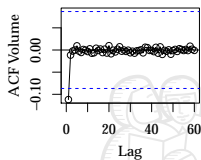
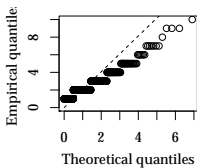
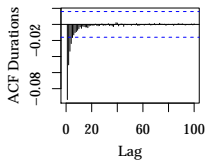
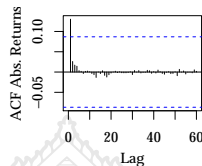
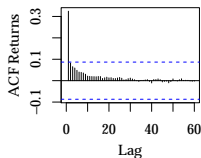
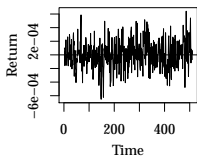
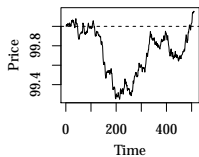
**EA** : fundamentalists and chartists with endogenous participation

parameter	value
$N$	1,000
$p^F$	100
tick	0.001
$h$	1,000
$p_0$	$p^F$

at the beginning of the simulation all chartists are provided a history of past prices between  $t = -h$  and  $t = 0$  that evolves (backwards) as a pure random walk

- irregular time series are pooled into minute-by-minute data
- results are averaged across 100 Montecarlo simulations

# NT scenario



▶ param

avg. # of trades = 14,958

$\kappa \approx 3.17$

▶ zoom



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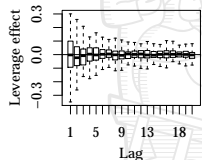
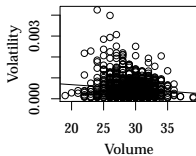
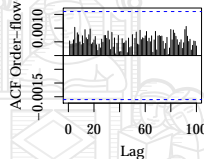
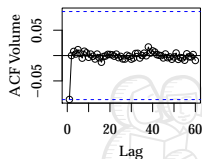
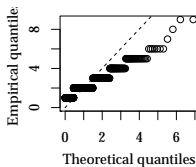
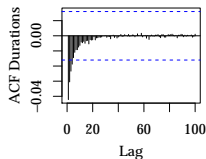
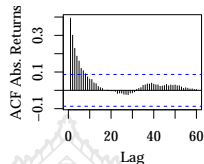
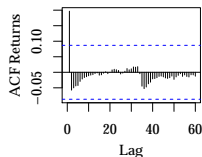
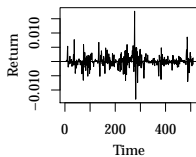
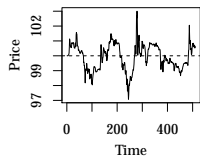
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# FC scenario



▶ param

avg. # of trades = 14,953

$\kappa \approx 14.5$

▶ zoom



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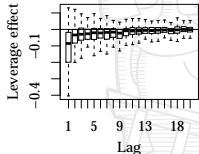
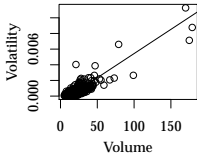
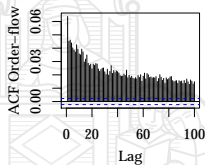
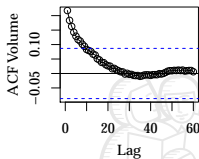
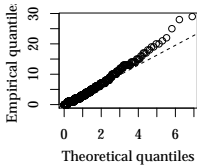
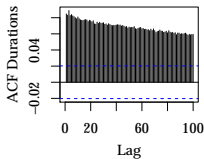
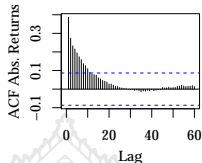
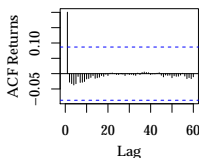
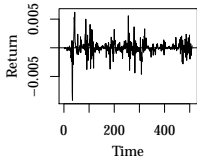
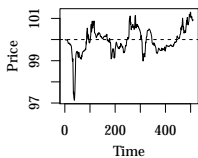
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# EA scenario



param

avg. # of trades = 9,991

$\kappa \approx 13.96$

zoom



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# Stylised facts

stylised fact	scenario			
	NT	FC	EA	
SF1	leptokurtic returns	X	✓	✓
SF2	no linear autocorr.	✓	✓	✓
SF3	volatility clustering	X	✓	✓
SF4	leverage effect	X	X	X
SF5	# price changes	✓	✓	✓
SF6	autocorr. durations	X	X	✓
SF7	fat-tailed durations	X	X	✓
SF8	order-flow clustering	X	X	✓
SF9	autocorr. volumes	X	X	✓
SF10	volume/volatility corr.	X	X	✓
SF11	U-shaped activity	X	X	X



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## assumptions

- no information diffusion – everything is common knowledge
- trading emerges as the consequence of differing (stable) beliefs
  - fundamentalists vs. chartists
- strict timing and microstructure from EURONEXT
- endogenous participation based on past volatility

## results

- NT slight dependence in returns quickly fading, # trades/day
- FC [NT]  $\oplus$  leptokurtosis and volatility clustering
- EA [FC]  $\oplus$  dependence in timing, volumes, and order-flow

## conceivable extensions

- time feedback in trading strategies
  - budget constraint/leverage requirement
  - more complex chartist specification  $\Rightarrow$  leverage effect
  - calibration of model parameters  $\Rightarrow$  policy experiments
- }  $\Rightarrow$  U-shaped seasonality

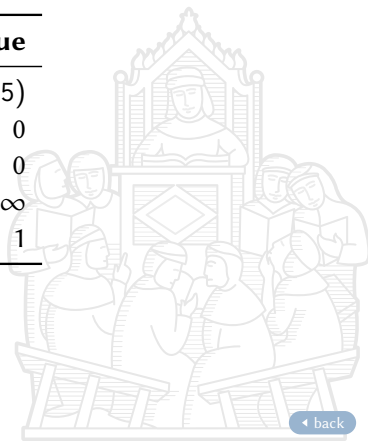
# Thank you very much!

[j.staccioli@sssup.it](mailto:j.staccioli@sssup.it)

## Dolphins project

...and props to the European Project 640772 - DOLFINS - H2020-FETPROACT-2014  
for financial support

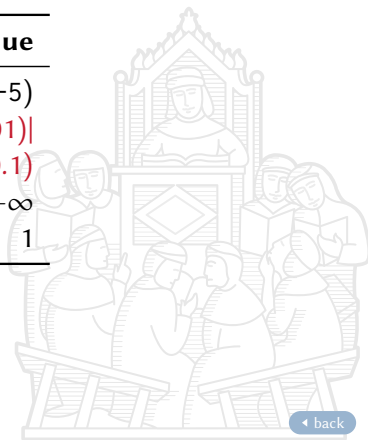
param.	value
$\varepsilon_t$	$\mathcal{N}(0, 5e-5)$
$w_i^F$	0
$w_i^C$	0
$\delta_t$	$+\infty$
$\phi$	1



◀ back



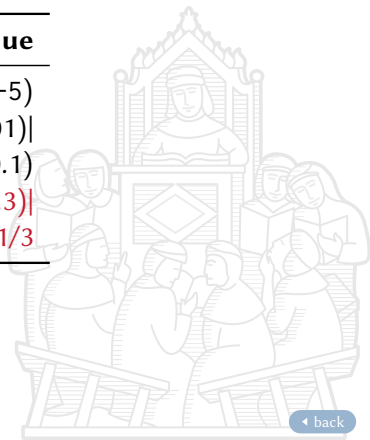
param.	value
$\varepsilon_t$	$\mathcal{N}(0, 5e-5)$
$w_i^F$	$ \mathcal{N}(0, 0.001) $
$w_i^C$	$\mathcal{N}(0.01, 0.1)$
$\delta_t$	$+\infty$
$\phi$	1



◀ back



param.	value
$\varepsilon_t$	$\mathcal{N}(0, 5e-5)$
$w_i^F$	$ \mathcal{N}(0, 0.001) $
$w_i^C$	$\mathcal{N}(0.01, 0.1)$
$\delta_t$	$ \mathcal{N}(0, 0.3) $
$\phi$	$1/3$



◀ back



# NT scenario (i)

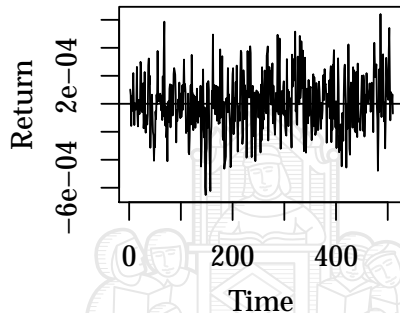
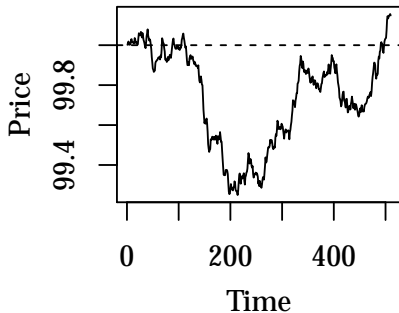


Figure: Price series (left) and return series (right) for a typical trading day

## NT scenario (ii)

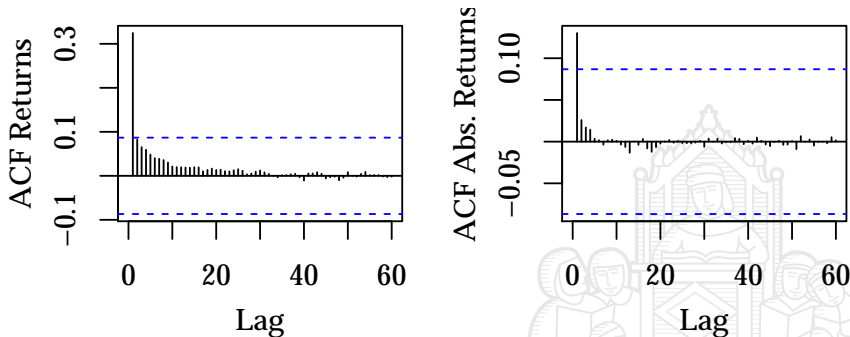


Figure: Autocorrelation of returns (left) and of absolute returns (right)





# NT scenario (iii)

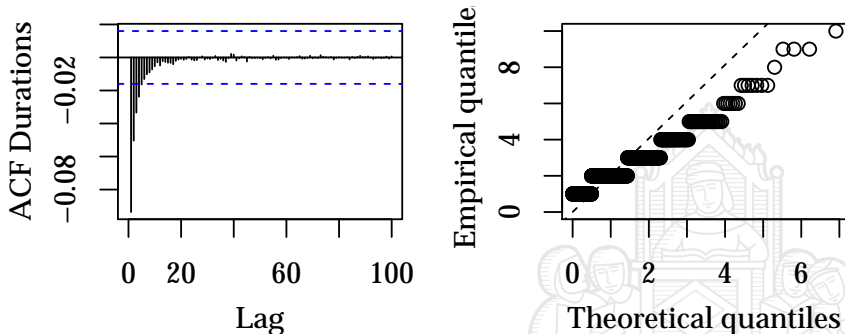


Figure: Autocorrelation of durations (left) and Q-Q of their distribution (right)

# NT scenario (iv)

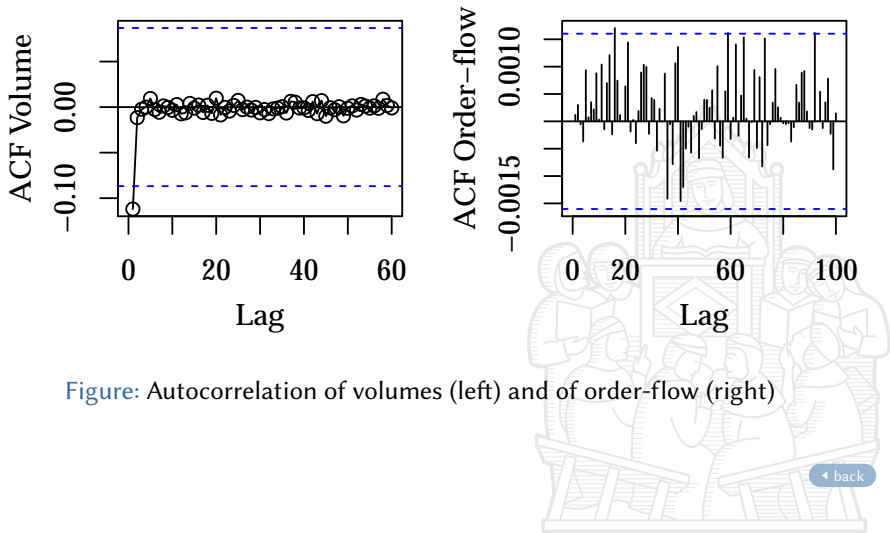


Figure: Autocorrelation of volumes (left) and of order-flow (right)

# NT scenario (v)

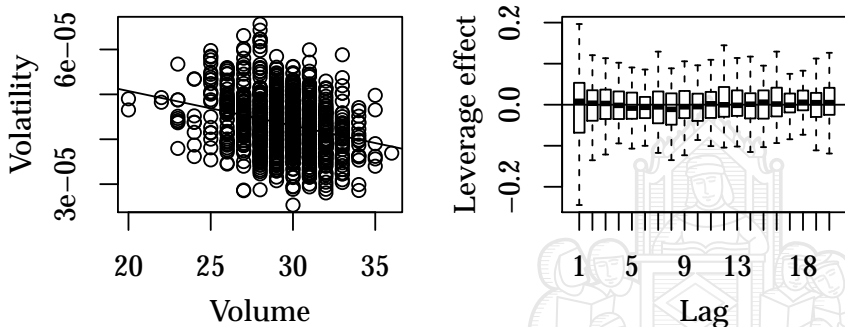


Figure: Volume/volatility correlation (left) and leverage effect (right)



# FC scenario (i)

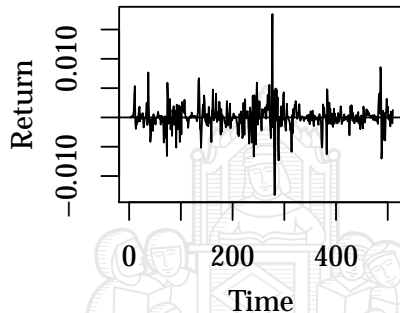
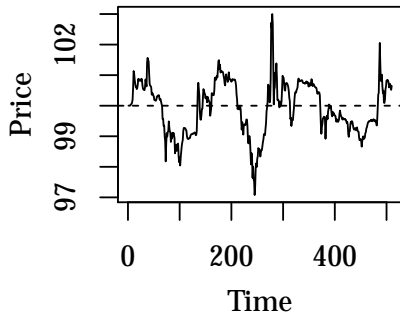


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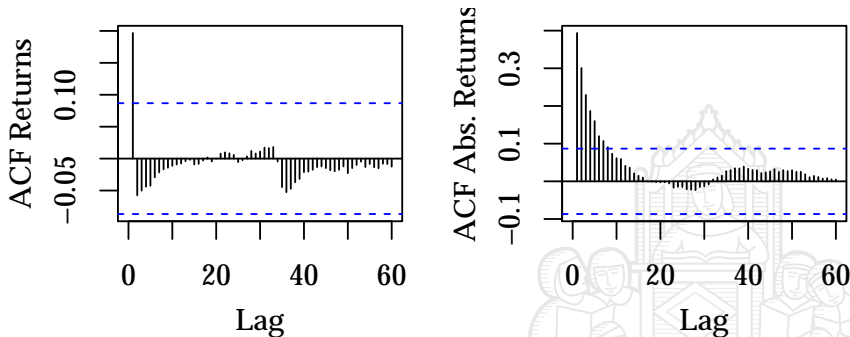


Figure: Autocorrelation of returns (left) and of absolute returns (right)

## FC scenario (iii)

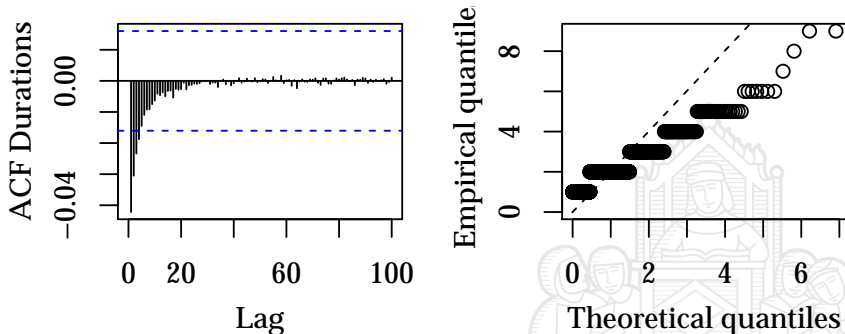


Figure: Autocorrelation of durations (left) and Q-Q of their distribution (right)

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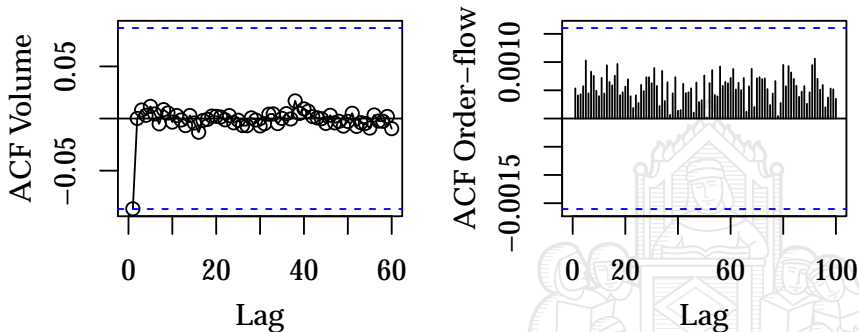


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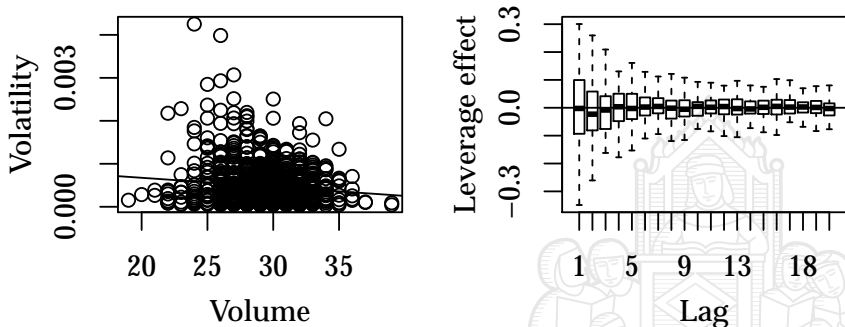


Figure: Volume/volatility correlation (left) and leverage effect (right)





# EA scenario (i)

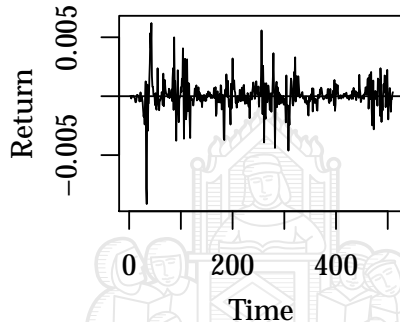
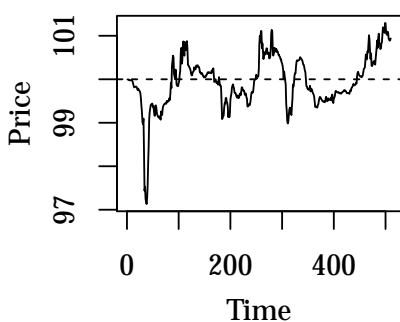


Figure: Price series (left) and return series (right) for a typical trading day

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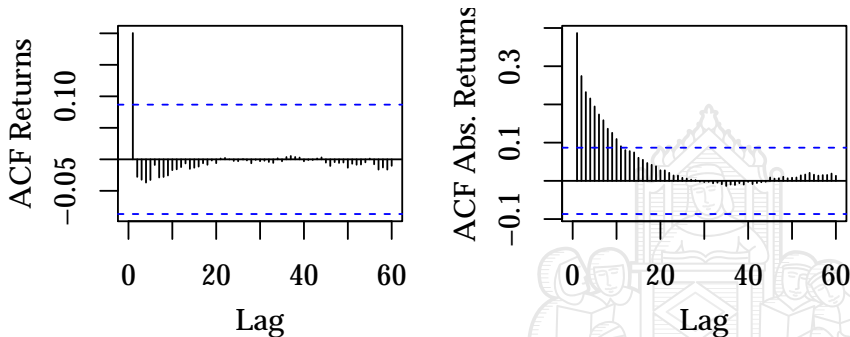


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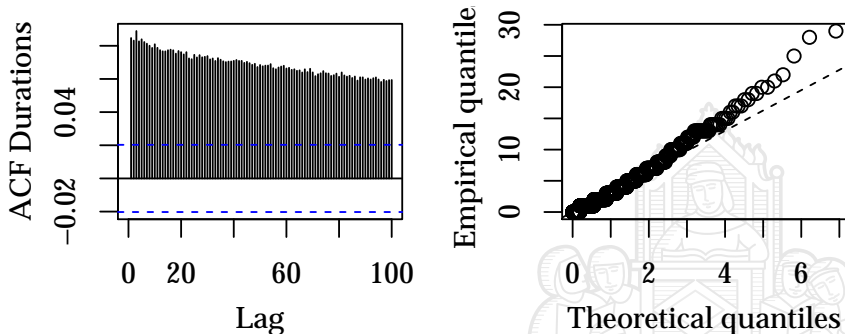


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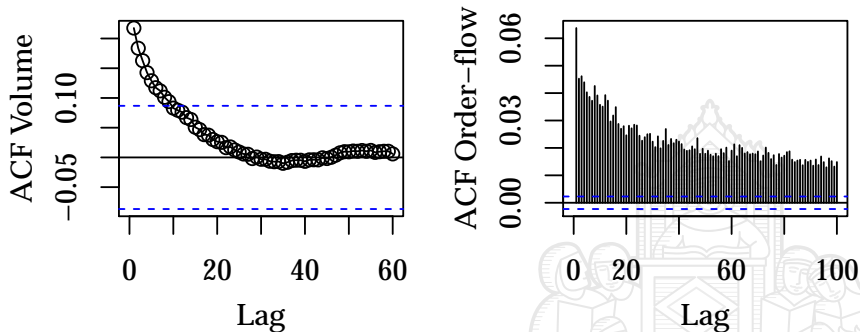


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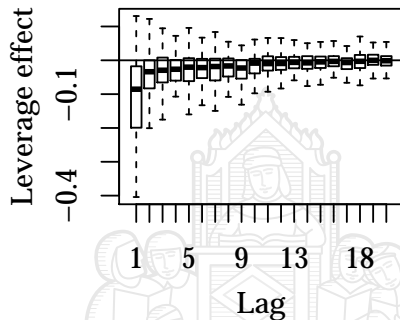
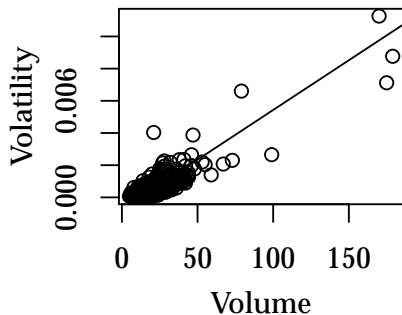


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