Bubble-and-bust dynamics under walrasian asset pricing and heterogeneous traders

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Bordeaux, 4th June, 2015
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2 Methodological perspective

3 The model

4 Simulation and results

5 Concluding remarks and conceivable extension
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Financial bubbles

Source: NASDAQ OMX Group

Shaded areas indicate US recessions - 2014 research.stlouisfed.org
“if the reason that the price is high today is only because investors believe that the selling price will be high tomorrow – when ‘fundamental’ factors do not seem to justify such a price – then a bubble exists”  

[Stiglitz, 1990]

“a sharp rise in the price of an asset or a range of assets in a continuous process, with the initial rise generating expectations of further rises and attracting new buyers – generally speculators interested in profits from trading in the asset rather than its use or earnings capacity”  

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Our proposal

We set up a model able to yield:

- *endogenous* bubble-and-bust dynamics
- as a result of the sole interaction among *heterogeneous adaptive traders*
- highlighting booms and crashes as intrinsic features of financial markets
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Heterogeneous Agents Models
- analytical investigations of the dynamical systems representing the laws of motion of the economy
- analytical tractability often leads to simplifying assumptions
- focus on asymptotic properties

Agent-Based Models
- computational (numerical) study of economies modelled as evolving systems of interacting agents
- complex behaviour specifications
- keep track of the whole dynamics
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The model

Consider a pure-exchange economy:

- $N$ heterogeneous traders (index $\mathcal{N} = \{1, \ldots, n, \ldots, N\}$);
- $L$ long-lived risky securities (index $\mathcal{L} = \{1, \ldots, \ell, \ldots, L\}$);
- a riskless bond;
- time is discrete (index $t \in \mathcal{T}$);
- risky securities, present in fixed amount, have ex-dividend price $p^\ell_t$ and pay random dividend $d^\ell_t$ at the end of each period;
- the bond, inelastically supplied, has price normalized to 1 (numéraire) and yields $r_f > 0$ in every $t$;
- trader wealth $W_{n,t}$ equals the market value of the portfolio he holds;
- fundamentals are proxied by the dividend yield $e^\ell_t = \frac{d^\ell_t}{p^\ell_{t-1}}$, $\forall \ell \in \mathcal{L}$. 
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Trader behaviour

At the beginning of each time step, trader \( n \) invests a share \( x_{n,t}^\ell \) of his wealth in security \( \ell \); the decision is made according to the information set

\[
\mathcal{I}_t = \{ p_1^\tau, \ldots, p_L^\tau; d_1^\tau, \ldots, d_L^\tau \mid \tau < t \}
\]

that is common knowledge, and to trader-specific investment function

\[
f_n : \mathbb{R}^{\tau \times L} \rightarrow \mathbb{R}^L \quad \text{such that} \quad x_{n,t} = f_{n,t}(\mathcal{I}_t)
\]

that is independent on wealth, coherent with CRRA attitude.

At every time step, each trader faces an optimisation problem of the form:

\[
\max_{x_{n,t}} \mathbb{E} \left[ \frac{W_{n,t}^{1-\gamma_n} - 1}{1 - \gamma_n} \right]
\]

s.t.

\[
W_{n,t} = W_{n,t-1} \cdot \left[ x_{n,t-1}^0 \cdot (1 + r_f) + \sum_{\ell=1}^L x_{n,t-1}^\ell \cdot \left( \frac{p_{t}^\ell}{p_{t-1}^\ell} + e_t^\ell \right) \right]
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where \( \gamma_n > 0 \) denotes the risk-aversion coefficient.
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At the beginning of each time step, trader $n$ invests a share $x^\ell_{n,t}$ of his wealth in security $\ell$; the decision is made according to the information set

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Trader expectations

We assume the trader forms expectations about future price returns and their (co)variances by means of EWMA predictors over the information set previously defined:

\[
\hat{\rho}_{n,t}^\ell = \lambda_n \cdot \sum_{\tau=0}^{\infty} (1 - \lambda_n)^\tau \cdot \rho_{t-\tau-1}^\ell
\]

\[
\hat{\sigma}_{\rho,n,t}^{\ell,h} = \lambda_n \cdot \sum_{\tau=0}^{\infty} (1 - \lambda_n)^\tau \cdot \left[ \rho_{t-\tau-1}^\ell - \hat{\rho}_{n,t-\tau-1}^\ell \right] \cdot \left[ \rho_{t-\tau-1}^h - \hat{\rho}_{n,t-\tau-1}^h \right]
\]

where \( \rho_t^\ell = \frac{p_t^\ell}{p_{t-1}^\ell} - 1 \) is the price return of security \( \ell \) between \( t - 1 \) and \( t \). The memory decay factor \( \lambda_n \in (0, 1) \) captures the way relative weights are distributed across more recent and older observations.
We adopt the same mean-variance approximation of the optimal investment function proposed in Chiarella and He (2001):

\[
x_{n,t} = \frac{1}{\gamma_n} \cdot C_{n,t}^{-1} \cdot \left[ E_{n,t} - r_f \cdot 1 \right]
\]

where \( E_{n,t} \) and \( C_{n,t} \) are, respectively, the vector of expected total returns and the expected variance-covariance matrix and \( d_n \) is a behavioural parameter:

- \( d_n = 0 \) trader \( n \) is a fundamentalist
- \( d_n > 0 \) trader \( n \) is a trend-chaser
- \( d_n < 0 \) trader \( n \) is a trend-contrarian

Assumption

\( e^\ell_t \) is drawn at each time step from a \( L \)-dimensional known probability distribution with mean \( \bar{e} \) and covariance matrix \( \Sigma \).
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Proposition

If short positions are not allowed, i.e.

\[ x_{n,t}^{\ell} \in (0, 1) \quad \forall n \in \mathcal{N}, \forall \ell \in \mathcal{L}, \forall t \in \mathcal{T} \]

then prevailing prices exist, are unique and strictly positive. It holds:

\[
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No trader can take short position in any asset, i.e. the image of traders’ investment functions is restricted such that

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Market selection and survival patterns

**Definition**

Individual wealth shares:

\[ \varphi_{n,t} = \frac{W_{n,t}}{\sum_{n=1}^{N} W_{n,t}} \]

- A trader \( n \) is said to ‘survive’ the economy if his long-run wealth-share is significantly different from 0, i.e. if \( \lim_{t \to \infty} \varphi_{n,t} > 0 \)

- A trader \( n \) is said to ‘dominate’ the economy if his long-run wealth-share is significantly close to 1, i.e. if \( \lim_{t \to \infty} \varphi_{n,t} = 1 \)

Following Anufriev et al. (2006), two types of equilibria are possible:

1. Single-survivor equilibria (most ‘aggressive’ trader)
2. Multiple-survivor equilibria (non-generic)
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<tr>
<td>Number of risky assets</td>
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</tr>
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<td>Static population</td>
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<td>Riskless rate of return</td>
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**Table:** Parameters and initial conditions (1)
Figure: Evolution of wealth-share for the least-risk-averse trader. Single-survivor.
### Simulation results - survival patterns (cont’d)

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**Table:** Parameters and initial conditions (2)
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### Table: Parameters and initial conditions (3)

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Simulation results - survival patterns (cont’d)

Figure: Multiple-survivor equilibrium. Evolution of wealth-shares.
Heterogeneity in the risk-aversion coefficient, within the stability domain of the system, triggers a wealth-driven selection mechanism.

- For a short memory-span (large $\lambda$), the least risk-averse trader survives and dominates the economy;
- For a long memory-span (small $\lambda$), multiple traders, still low-risk averse, survive and display identical investment decisions.

Out of the stability domain of the system (i.e. for large enough $\lambda$, following Anufriev et al. 2006) selection does not occur: individual wealth-shares keep fluctuating indefinitely.
Heterogeneity in the risk-aversion coefficient, within the stability domain of the system, triggers a wealth-driven selection mechanism.

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## Simulation results: price dynamics

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial population size</td>
<td>$N = 1000$</td>
</tr>
<tr>
<td>Number of risky assets</td>
<td>$L = 1$</td>
</tr>
<tr>
<td>Static population</td>
<td>true</td>
</tr>
<tr>
<td>Riskless rate of return</td>
<td>$r_f = 0.02$</td>
</tr>
<tr>
<td>$\gamma$ distribution</td>
<td>$\gamma_n \sim U(1.0, 500.0)$</td>
</tr>
<tr>
<td>$\lambda$ distribution</td>
<td>$\lambda_n = 0.0036$, $\forall n \in \mathcal{N}$</td>
</tr>
<tr>
<td>$d$ distribution</td>
<td>$d_n = 1.0$, $\forall n \in \mathcal{N}$</td>
</tr>
<tr>
<td>Initial wealth endowment</td>
<td>$W_{n,0} = 50.0$, $\forall n \in \mathcal{N}$</td>
</tr>
<tr>
<td>Yield mean</td>
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</tr>
<tr>
<td>Yield variance</td>
<td>$\sigma^2_e = 1.0e^{-4}$</td>
</tr>
<tr>
<td>Yield realisation distribution</td>
<td>$e_t \sim \mathcal{N}(\bar{e}, \sigma^2_e)$</td>
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<tr>
<td>Initial risky asset price level</td>
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<tr>
<td>$x_n$ admissible interval</td>
<td>$x_{n,t} \in [0.01, 0.99]$, $\forall n \in \mathcal{N}$, $\forall t$</td>
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**Table:** Parameters and initial conditions (4)
Figure: Price dynamics. Smooth and monotone convergence to equilibrium.
Figure: Price dynamics. Emergence of a bubble-and-bust cycle. $\lambda_n = 0.00365$
Simulation results: price dynamics (cont’d)

Figure: Price dynamics. Emergence of multiple bubble-and-bust cycles. $\lambda_n = 0.155$
Simulation results: price dynamics (cont’d)

Figure: Price dynamics. No convergence to an equilibrium value. $\lambda_n = 0.16$
The emergent properties observed in a trend-chasers-only setting maintain robustness with respect to the introduction of fundamentalist and trend-contrarian traders in the economy.

- Fundamentalists are expected to stabilise the price as they act against chartists whenever current price deviates from its fundamental value;
- Contrarians shall counteract the attempt made by trend-chasers to exacerbate the price trend by acting in a symmetrical fashion.

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We now shift the analysis to the transitional price dynamics:

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**Table:** Parameters and initial conditions (5)
Simulation results: price dynamics (cont’d)

Figure: Price dynamics. Fundamentalists vs. chartists.
Simulation results: price dynamics (cont’d)

![Price dynamics](image)

**Figure:** Price dynamics. Fundamentalists vs. chartists. $\gamma_n \sim \mathcal{U}(1.0, 1000.0)$
Simulation results: micro-failure

Figure: Price dynamics. Micro-failure striking every $\tau = 15$ periods.
Concluding remarks

1. Analysis of individual wealth-shares
   - strong market selection mechanism
   - single- and multiple-survivor equilibria
   - riskier investment functions globally dominate

2. Analysis of transitional price dynamics
   - emergence of bubble-and-bust cycles
   - robust to the introduction of fundamentalist and trend-contrarian

Our framework can be extended in a number of directions:
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   - dynamic population
   - more realistic traders’ behaviour (prospect theory, herding) and learning (genetic algorithms, classifier systems)
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