Wealth-driven asymptotic survival in a financial market with demand shocks

Pietro Dindo[†] Jacopo Staccioli[‡]

[†] *Department of Economics - University of Venice*

[‡] *Institute of Economics - Scuola Superiore Sant'Anna, Pisa*

Society for Computational Economics

CEF — Computing in Economics and Finance

Bordeaux, 28th June, 2016

Outline

Context and motivation

Financial markets

- largely miscellaneous ecology of traders
- all seeking for some sort of *'return'*
- but strategies are likely to differ

Example: pension fund vs. hedge fund

- different preferences (risk attitude, time horizon...)
- different regulatory requirement (max leverage, liquidity...)

Reference literature

- bounded rationality
- heterogeneous agents models (HAMs)

Context and motivation (cont'd)

Usual classification

- chartist vs. fundamentalist

Our approach: riskiness of the portfolio (1 risky stock, 1 risk-free bond)

- higher risk, higher return
- higher risk, higher vulnerability to liquidity shock

Example: fixed-mix strategy

low-risk portf. use bond as collateral (no recomposition)

high-risk portf. sell stock, purchase bond and use as collateral (fire sale)

Our classification

- lion vs. sheep, daredevil vs. prudent, aggressive vs. shy, hawk vs. dove, short vs. long horizon etc…

Our contribution

- explore this trade-off
- wealth-driven market selection
- conditions for survival and dominance

technically stochastic vs. constant fixed-mix strategies

Main findings

- long-run heterogeneity
- non-trivial price-wealth dynamics
- volatility clustering

Outline

The model

Consider a stylised financial market (no consumption)

- trade takes place in discrete time t
- risk-free bond
  - inelastic supply
  - constant r_f return (zero in the rescaled model)
  - price normalised to 1 (*numéraire*)
- long-lived risky security
  - non-negative dividend d_t
  - unitary constant supply
  - market clearing price p_t

At each time step t trader n

- invests a fraction $x_{n,t}$ of her wealth $w_{n,t}$ into the risky security
- residually invests $(1 - x_{n,t}) \cdot w_{n,t}$ into the bond

Trader n wealth $w_{n,t}$ equals the current market value of her portfolio

The model (cont'd)

Assumption

Dividend grows geometrically

$$d_t = d_{t-1} \cdot (1 + g) \qquad g > 0 = r_f$$

Definitions

- dividend yield of the risky asset

$$e_t := \frac{d_t}{p_{t-1}} = e_{t-1} \frac{1+g}{1+r_{t-1}}$$

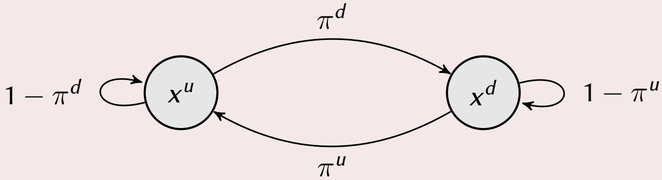
where $r_t := \frac{p_t}{p_{t-1}} - 1$ is the net return

- wealth share of agent n

$$\varphi_{n,t} := \frac{w_{n,t}}{\sum_n w_{n,t}}$$

Assumption

- constant trader (sheep) always invests $\overline{x} \in (0, 1)$
- stochastic trader (lion) invests according to Markov process



$$\pi^u > 0 \qquad \pi^d > 0 \qquad 0 < x^d < x^u < 1$$

Laws of motion of the economy

$$\mathcal{F}_{x_{t-1},x_t} : \mathcal{D} \to \mathcal{D}, \qquad \mathcal{D} = \Delta \times (-1, +\infty) \times \mathbb{R}_+$$

$$\begin{cases} \varphi_t = \varphi_{t-1} \dfrac{1 + x_{t-1}(r_t + e_t)}{1 + (r_t + e_t)\left[\varphi_{t-1} x_{t-1} + (1 - \varphi_{t-1})\overline{x}\right]} \\[2ex] r_t = \dfrac{\varphi_{t-1}\left[x_t(1 + e_t x_{t-1}) - x_{t-1}\right] + (1 - \varphi_{t-1}) e_t \overline{x}^2}{\varphi_{t-1} x_{t-1}(1 - x_t) + (1 - \varphi_{t-1})\overline{x}(1 - \overline{x})} \\[2ex] e_t = e_{t-1} \dfrac{1 + g}{1 + r_{t-1}} \end{cases}$$

where φ_t denotes the wealth share of the stochastic trader

Outline

Long-run outcomes

Terminology

- trader n is said to *survive* on $\{x\}$ if $\limsup_{t\to\infty} \varphi_{n,t} > 0$
- trader n is said to *vanish* on $\{x\}$ if $\limsup_{t\to\infty} \varphi_{n,t} = 0$
- trader n is said to *dominate* on $\{x\}$ if $\liminf_{t\to\infty} \varphi_{n,t} = 1$

Proposition

If $(\varphi^*, r^*, e^*) \in \mathcal{D}$ is a fixed point of system $\mathcal{F}$ then either

1. the constant trader dominates (i.e. the stochastic trader vanishes)
2. the stochastic trader dominates (i.e. the constant trader vanishes)

A dominant sheep

$$\varphi^{\mathfrak{S}} = 0 \qquad r^{\mathfrak{S}} = g \qquad e^{\mathfrak{S}} = g\frac{1-\overline{x}}{\overline{x}}$$

Proposition

Steady state $\mathfrak{S}$ is locally asymptotically stable if

$$\lambda^{\mathfrak{S}} = \frac{\left(\overline{x} + gx^u\right)^{\frac{\pi^u}{\pi^u+\pi^d}}\left(\overline{x} + gx^d\right)^{\frac{\pi^d}{\pi^u+\pi^d}}}{\overline{x}(1+g)} < 1$$

- $\overline{x} \geq x^u \implies \mathfrak{S}$ is locally asymptotically stable
- $\overline{x} \leq x^d \implies \mathfrak{S}$ is unstable

A dominant sheep (cont'd)

Proposition

$\exists!\ x' \in (0,1)$ such that

- $\forall\ \overline{x} > x'$, $\mathfrak{S}$ is locally asymptotically stable
- $\forall\ \overline{x} < x'$, $\mathfrak{S}$ is unstable

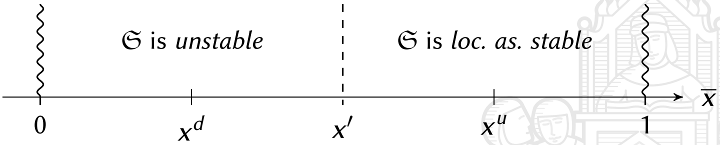
Moreover $x^d < x' < x^u$

Special case

$\pi^u = \pi^d \implies \mathfrak{S}$ is locally asymptotically stable if

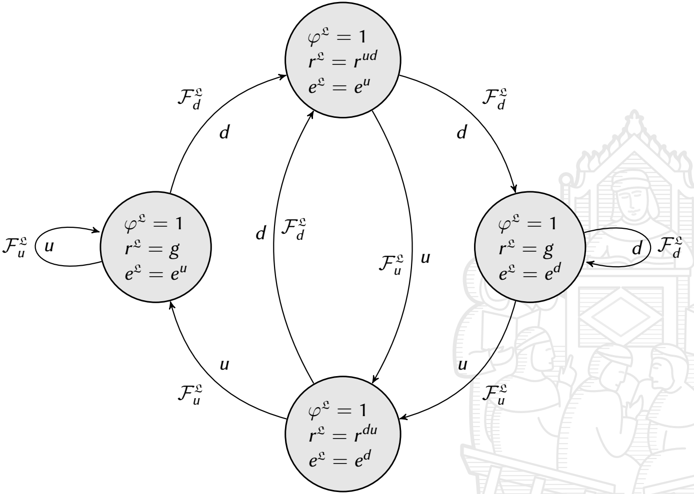
$$\overline{x} > \frac{x^u + x^d}{2} - O(g) \quad \text{as} \quad g \to 0^+$$

Fact $\exists\ \varepsilon > 0 : |\pi^u - \pi^d| < \varepsilon \implies \overline{x} > \mathbb{E}[x_t] - O(g) \quad \text{as} \quad g \to 0^+$



$\mathfrak{S}$ is *unstable*
$\mathfrak{S}$ is *loc. as. stable*
0
x^d
x^l
x^u
1
$\overline{x}$

A dominant lion



It holds $r^{ud} < g < r^{du}$ and $e^{u} < e^{d}$. Moreover, *'usually'* $r^{ud} < 0$

A dominant lion (cont'd)

Proposition

Steady state $\mathfrak{L}$ is locally asymptotically stable if

$$\lambda^{\mathfrak{L}} = \frac{1}{1+g}\left[1+\frac{g\bar{x}}{x^u}\right]^{\frac{\pi^u(1-\pi^d)}{\pi^u+\pi^d}}$$

$$\cdot\left[\frac{\bar{x}\left[g(1-x^u)-(x^u-x^d)\right]+x^u(1-x^d)}{x^u(1-x^u)}\cdot\frac{\bar{x}\left[g(1-x^d)+(x^u-x^d)\right]+x^d(1-x^u)}{x^d(1-x^d)}\right]^{\frac{\pi^u\pi^d}{\pi^u+\pi^d}}$$

$$\cdot\left[1+\frac{g\bar{x}}{x^d}\right]^{\frac{\pi^d(1-\pi^u)}{\pi^u+\pi^d}} < 1$$

A dominant lion (cont'd)

Proposition

$\exists!\ x'' \in (0,1)$ such that

- $\forall\ \bar{x} < x''$, $\mathfrak{L}$ is locally asymptotically stable
- $\forall\ \bar{x} > x''$, $\mathfrak{L}$ is unstable

Sufficient conditions

- if $\bar{x} \leq x^d$ and $g \geq \dfrac{x^u - x^d}{1 - x^u}$ with at least one strict inequality sign, then steady state $\mathfrak{L}$ is locally asymptotically stable
- if $\bar{x} \geq x^d$ and $g \leq \dfrac{x^u - x^d}{1 - x^u}$ with at least one strict inequality sign, then steady state $\mathfrak{L}$ is unstable

Proposition

$\exists\, \hat{g} > 0$ such that $\forall\, g < \hat{g}$ it holds $x'' < x^d < x'$. In particular

$$\hat{g} = \frac{x^u - x^d}{1 - x^u}$$

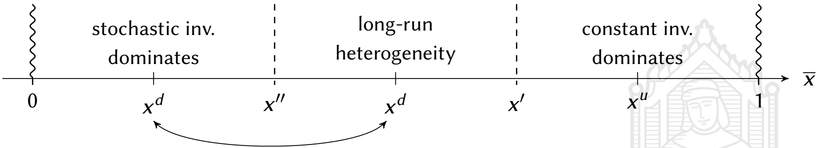
Therefore there *generically* exists a non-degenerate interval (x'', x') such that both $\mathfrak{S}$ and $\mathfrak{L}$ are unstable $\forall\, \overline{x} \in (x'', x')$

Fact

A numerical inspection of the parameter space reveals that $x'' < x'$

- $\forall\, x^u, x^d, \pi^u, \pi^d \in \{0.01, 0.02, \ldots, 0.99\}$ such that $x^d < x^u$
- $\forall\, g = \hat{g} \cdot 10^k, k \in \mathbb{N}_+$ such that $g \leq 10$

Long-run heterogeneity (cont'd)



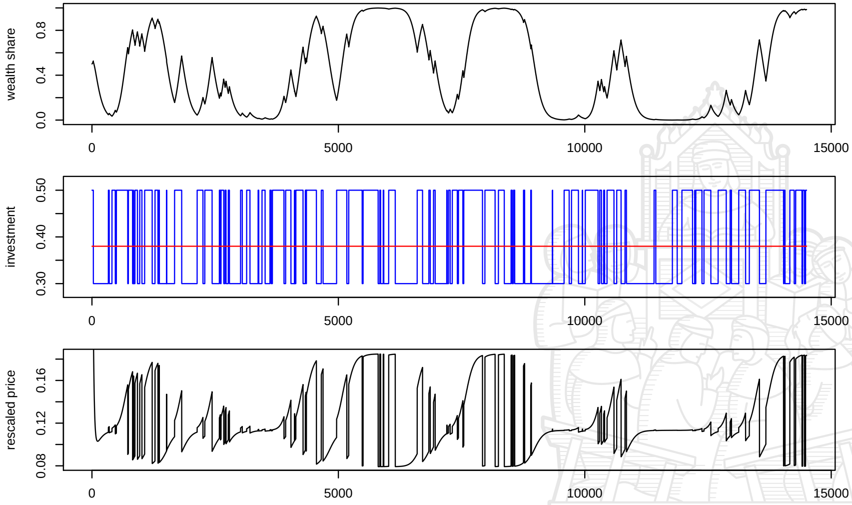
Corollary

When g is small the constant trader is able to *invade* the stochastic trader even adopting an always strictly safer position $\overline{x} < x^d < x^u$

Outline

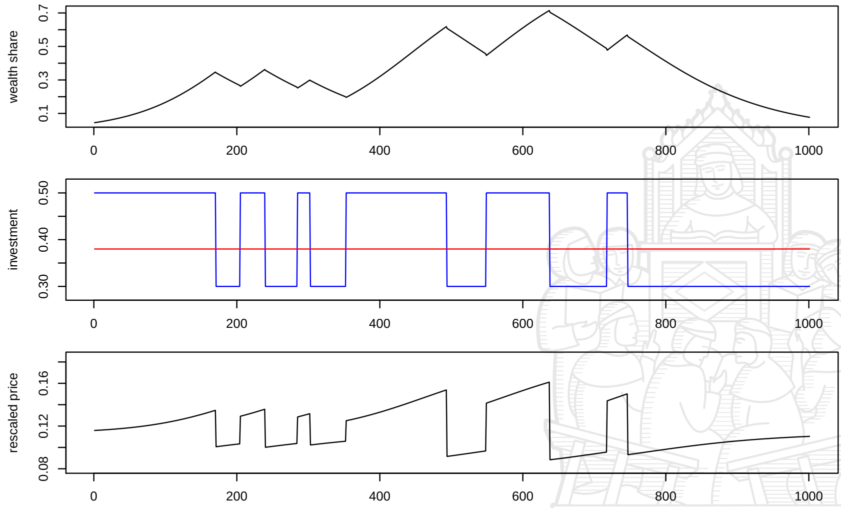
Initialisation

Description	Variable	Value
dividend rate of growth	g	0.05
constant investment	$\overline{x}$	0.38
stochastic investment up	x^u	0.5
stochastic investment down	x^d	0.3
probability down when up	π^d	0.01
probability up when down	π^u	0.01
initial yield	e_0	0.01
initial return	r_0	0.0
initial wealth share	φ_0	0.5



wealth share
0.0
0.4
0.8
0
5000
10000
15000
investment
0.30
0.40
0.50
0
5000
10000
15000
rescaled price
0.08
0.12
0.16
0
5000
10000
15000

Simulation close-up



Outline

Concluding remarks

Main findings

- higher risk $\implies$ higher return, but only locally!
- usually $\exists$ intrinsic penalty in adopting stochastic strategy
- a sheep can invade a lion with an always-strictly-safer portfolio
- the lion (fire-)sells at low price and buys at high price
- generic long-run heterogeneity
- endogenous fluctuations of price and wealth
- volatility clustering

Concluding remarks (cont'd)

Conceivable extensions

- arbitrary number of risky security
- *stochastic g*
- more complicated portfolio responses (e.g. staircase)
- propagation of shock *within* traders' class (industry)

Hope you slept comfortably

Thank you very much!

j.staccioli@sssup.it

…and props to the European Project PIOF-GA-2011-300637 MSAEO for financial support