Endogenous financial bubbles: an Agent-Based approach

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Outline

What is a financial bubble?

Many definitions have been given; among the others:

if the reason that the price is high today is only because investors believe that the selling price will be high tomorrow – when 'fundamental' factors do not seem to justify such a price – then a bubble exists (Stiglitz, 1990);

a sharp rise in the price of an asset or a range of assets in a continuous process, with the initial rise generating expectations of further rises and attracting new buyers – generally speculators interested in profits from trading in the asset rather than its use or earnings capacity (Kindleberger, The New Palgrave Dictionary of Economics);

What is a financial bubble?

when the prices of securities or other assets rise so sharply and at such a sustained rate that they exceed valuations justified by fundamentals, making a sudden collapse likely – at which point the bubble bursts (FT);

when the price of something does not equal its market fundamentals for some period of time for reasons other than random shocks (Rosser, 2000);

. . . many other definitions. . .

For a complete review of bubble definitions refer to Siegel (2003).

Historical evidence

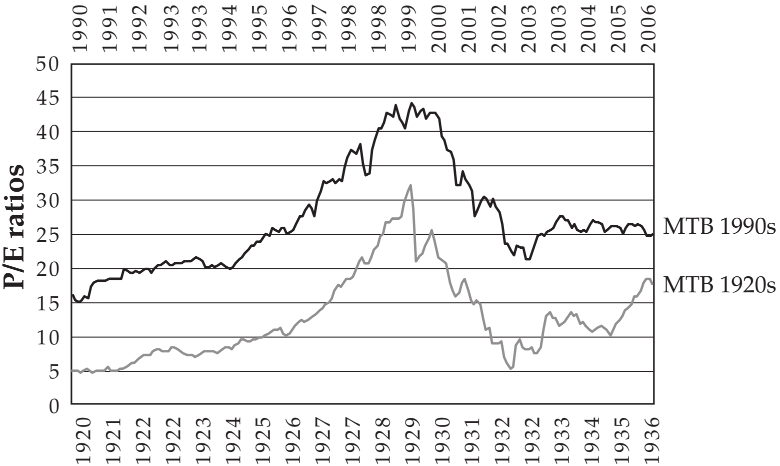

Figure : Shiller P/E ratios 1920 – 1936 and 1990 – 2006. MTB stands for Major Technological Bubble. Source: Shiller (2005) and Perez (2009).

Historical evidence

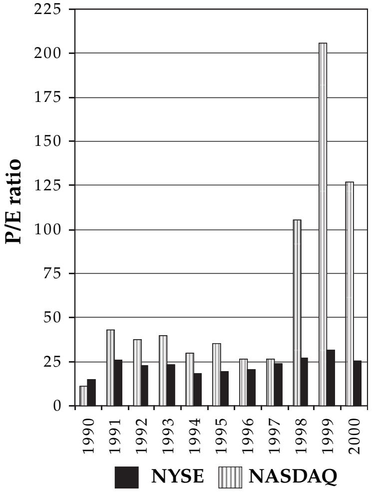

Figure : P/E ratios NYSE – NASDAQ, 1990 – 2000. Source: Perez (2009).

Historical evidence

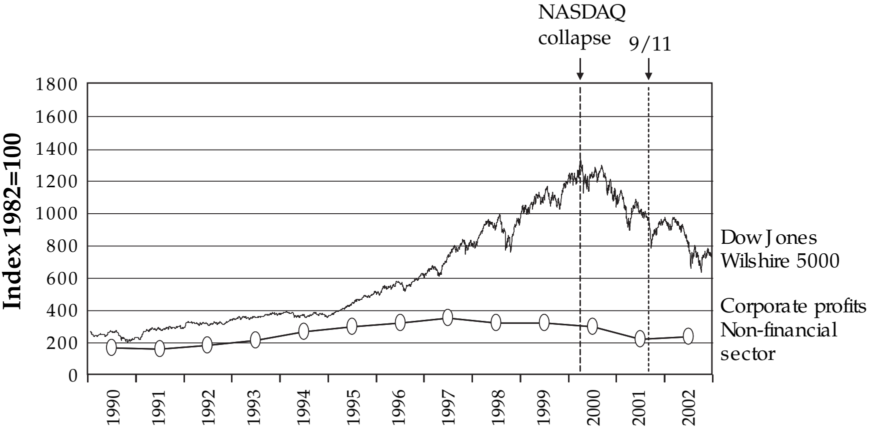

Figure : Evolution of annual corporate profits for non-financial sector and Daily DowJones Wilshire 5000 USA 1990 – 2002. Source: Perez (2009).

The interpretative framework

- **Greater fool**: under asymmetric information, every agent knows that prices exceed fundamental value but not everyone knows that all other agents are aware of it (Kindleberger, 1978).
- **Short-selling restrictions**: for overvalued assets, new piece of information is not immediately incorporated into market price (Shleifer and Vishny, 1997; Hong et al, 2006).
- **Noise traders**: take decisions with no reference to fundamental data (Black, 1986; De Long et al, 1990).

The interpretative framework

- **Momentum-Overreaction**: traders try to discover trends and thereby overestimate the autocorrelation even in random series (Offerman and Sonnemans, 2004).
- **Herd behaviour**: informational cascades (Banerjee, 1992; Welch, 1992); reputational herding (Scharfstein and Stein, 1990); investigative herding (Brennan, 1990).
- **Technology**: over-optimism on technological and market opportunities especially associated with newly emerging technological paradigms (Perez 2002, 2009, Shiller 2005).

Our proposal

We propose a model that captures:

1. Self-reinforcing departures of asset returns from dividend dynamics;
2. Excess volatility and excess cross-correlations of returns;
3. Burst processes triggered by *small events* (e.g. micro-failures) that become relevant when asset prices are decoupled from fundamentals;
4. Financial innovations which are understood to diversify risk, ultimately aggregate it and transform *idiosyncratic* into *systemic* risk. [we provide just an intuition]

The model

Consider a pure-exchange financial economy:

- N heterogeneous traders;
- L long-lived risky securities $(A^1, \ldots, A^\ell, \ldots, A^L)$;
- a riskless bond B;
- time is discrete;
- risky securities, present in fixed amount, have ex-dividend price p_t^ℓ and pay a random dividend d_t^ℓ at the end of each period;
- the bond, inelastically supplied, have price normalized to 1 (*numéraire*) and yields $r_f > 0$ in every t;
- trader wealth is the market value of the portfolio he holds:

$$W_{n,t} = \sum_{\ell=1}^{L} A_{n,t}^\ell \cdot p_t^\ell + B_{n,t}$$

Trader behaviour

At the beginning of each time step, trader n invests a share $x^{\ell}_{n,t}$ of his wealth in security ℓ; the decision is made according to the information set

$$\mathcal{I}_t = \{p^1_\tau, \ldots, p^L_\tau;\ d^1_\tau, \ldots, d^L_\tau \mid \tau < t\}$$

that is common knowledge, and to trader-specific investment function

$$\mathbf{x}_{n,t} = f_{t,n}(\mathcal{I}_t) \quad \text{such that} \quad \mathbf{x}_{n,t} \cdot \mathbf{1} \leq 1$$

that is independent on wealth, coherently with CRRA preferences.
The amount of wealth invested in the bond is residually determined:

$$x^0_{n,t} = 1 - \sum_{\ell=1}^{L} x^{\ell}_{n,t}$$

Demand and equilibrium condition

Individual excess demand for asset ℓ holds:

$$\Delta^{\ell}_{n,t}(\mathbf{p}) = \frac{x^{\ell}_{n,t} \cdot B_{n,t-1}}{p^{\ell}} - (1 - x^{\ell}_{n,t}) \cdot A^{\ell}_{n,t-1} + x^{\ell}_{n,t} \cdot \sum_{h \neq \ell} A^{h}_{n,t-1} \cdot \frac{p^{h}}{p^{\ell}}$$

An idealized auctioneer computes the aggregate excess demand for every asset ℓ and announces the price p^{ℓ}_{t} equating it to zero:

$$\sum_{n=1}^{N} \Delta^{\ell}_{n,t}(\mathbf{p}) = 0 \qquad \forall \ell = 1, \ldots, L$$

$$\sum_{h=1}^{L} p^{h} \cdot (\delta_{\ell,h} - \sum_{n=1}^{N} x^{\ell}_{n,t} \cdot A^{h}_{n,t-1}) = \sum_{n=1}^{N} x^{\ell}_{n,t} \cdot B_{n,t-1} \qquad \forall\ \ell$$

where $\delta_{\ell,h}$ is the Kronecker delta function.

Pricing of risky assets

Proposition

If short positions are not allowed, i.e.

$$x_{n,t}^{\ell} \in (0,1) \qquad \forall\ n = 0, \ldots, N,\ \forall\ \ell = 0, \ldots, L,\ \forall\ t$$

then prevailing prices exist, are unique and strictly positive.

Proof.

See the paper or refer to Anufriev et al (2011) for an analogous result. $\square$

Useful additional definitions

Dividend yield: the ratio of dividend over past realized price

$$e_t^\ell = \frac{d_t^\ell}{p_{t-1}^\ell}$$

Aggregate wealth: the sum of all individual wealth levels

$$W_t = \sum_{n=1}^{N} W_{n,t}$$

Individual wealth shares: the ratios of each individual wealth out of aggregate wealth

$$\varphi_{n,t} = \frac{W_{n,t}}{W_t}$$

Market portfolio: the wealth-weighted sum of individual portfolios

$$\mathbf{x}_t = \sum_{n=1}^{N} \mathbf{x}_{n,t} \cdot \varphi_{n,t}$$

The overall dynamics

From the above **Proposition**, aggregate wealth and prevailing prices are determined:

$$W_t = W_{t-1} \cdot \frac{x^0_{t-1} \cdot (1 + r_f) + \sum_{\ell=1}^{L} x^\ell_{t-1} \cdot e^\ell_t}{x^0_t}$$

$$p^\ell_t = p^\ell_{t-1} \cdot \frac{x^\ell_t}{x^\ell_{t-1}} \cdot \frac{x^0_{t-1} \cdot (1 + r_f) + \sum_{\ell=1}^{L} x^\ell_{t-1} \cdot e^\ell_t}{x^0_t}$$

Holdings are then updated according to new prices and payed dividends:

$$A^\ell_{n,t} = A^\ell_{n,t-1} + \Delta^\ell_{n,t}(\mathbf{p_t})$$

$$B_{n,t} = \left[B_{n,t-1} - \sum_{\ell=1}^{L} \Delta^\ell_{n,t}(\mathbf{p_t}) \cdot p^\ell_t \right] \cdot (1 + r_f) + \sum_{\ell=1}^{L} A^\ell_{n,t} \cdot d^\ell_t$$

The overall dynamics

The evolution of individual wealth holds:

$$W_{n,t} = W_{n,t-1} \cdot \left[x^0_{n,t-1} \cdot (1 + r_f) + \sum_{\ell=1}^{L} x^\ell_{n,t-1} \cdot \left(\frac{p^\ell_t}{p^\ell_{t-1}} + e^\ell_t \right) \right]$$

$$= W_{n,t-1} \cdot \left[x^0_{n,t-1} \cdot (1 + r_f) + \mathbf{x}_{n,t-1} \cdot \mathbf{e}_{n,t} + \sum_{\ell=1}^{L} x^\ell_{n,t-1} \cdot \frac{x^\ell_t}{x^\ell_{t-1}} \cdot \frac{W_t}{W_{t-1}} \right]$$

Individual wealth shares read:

$$\varphi_{n,t} = \frac{\varphi_{n,t-1} \left(x^0_{n,t-1} \cdot (1 + r_f) + \mathbf{x}_{n,t-1} \cdot \mathbf{e}_t \right)}{\sum_m \varphi_{m,t-1} \left(x^0_{m,t-1} \cdot (1 + r_f) + \mathbf{x}_{m,t-1} \cdot \mathbf{e}_t \right)}$$

$$+ \sum_{\ell=1}^{L} \varphi_{n,t-1} \cdot x^\ell_{n,t-1} \cdot \frac{\sum_m \varphi_{m,t} \cdot x^\ell_{m,t}}{\sum_m \varphi_{m,t-1} \cdot x^\ell_{m,t-1}}$$

Assumptions

We now make the following additional assumptions:

- The vector of stochastic yields $\mathbf{e}_t$ is randomly drown from a stationary L-dimensional distribution with support on positive real numbers, mean value $\bar{\mathbf{e}}$ and positive-definite variance-covariance matrix Σ. Both $\bar{\mathbf{e}}$ and Σ are common knowledge among all traders.
- Individual investment functions satisfy the conditions:

$$f_{n,t}^{\ell} > 0 \qquad \text{and} \qquad \sum_{\ell=0}^{L} f_{n,t}^{\ell} < 1$$

so that the above **Proposition** applies.

Trader optimization problem

At every time step, each trader faces an optimisation problem of the form:

$$\max_{\mathbf{x}_t} \; \mathbb{E}[U(W_t)]$$

s.t.

$$W_t = W_{t-1} \cdot \left[x_{t-1}^0 \cdot (1 + r_f) + \sum_{\ell=1}^{L} x_{t-1}^{\ell} \cdot \left(\frac{p_t^{\ell}}{p_{t-1}^{\ell}} + e_t^{\ell} \right) \right]$$

For notational convenience the n subscript is dropped here. Coherently with CRRA attitude, the utility function reads:

$$U(W_t) = \frac{W_t^{1-\gamma} - 1}{1 - \gamma}$$

where γ denotes the risk aversion coefficient.

Trader UMP - Expectation

We assume the trader forms expectations about future price returns and their (co)variances according to EWMA estimators over the information set previously defined:

$$\hat{\rho}_t^{\ell} = \lambda \cdot \sum_{\tau=0}^{\infty} (1-\lambda)^{\tau} \cdot \rho_{t-\tau-1}^{\ell}$$

$$\hat{\sigma}_{\rho t}^{\ell h} = \lambda \cdot \sum_{\tau=0}^{\infty} (1-\lambda)^{\tau} \cdot \left[\rho_{t-\tau-1}^{\ell} - \hat{\rho}_{t-\tau-1}^{\ell}\right] \cdot \left[\rho_{t-\tau-1}^{h} - \hat{\rho}_{t-\tau-1}^{h}\right]$$

where $\rho_t^{\ell} = \frac{p_t^{\ell}}{p_{t-1}^{\ell}} - 1$ is the price return of security ℓ between $t-1$ and t.
The decay factor λ captures how the relative weights are distributed across more recent and older observations.

Trader UMP - Investment function

Since an explicit solution of the UMP cannot be derived analytically, we use a mean-variance approximation of the optimal investment function (see Chiarella and He, 2001):

$$f^{\ell}\left(\mathcal{I}_t\right) = x_t^{\ell} = \frac{1}{\gamma} \cdot \hat{\mathbb{C}}^{-1} \cdot \left[\mathbb{E}\left(\boldsymbol{\rho}_t + \mathbf{e}_t\right) - r_f \cdot \mathbf{1}\right]$$

where $\hat{\mathbb{C}}$ is the expected covariance matrix of total returns. Still in line with previous literature we adopt the following expressions:

$$\mathbb{E}\left(\rho_t^{\ell} + e_t^{\ell}\right) = \delta + d \cdot \left(\hat{\rho}_t^{\ell} + \overline{e}^{\ell}\right)$$
$$\hat{\mathbb{C}}_{\ell,h} = \hat{\sigma}_{\rho t}^{\ell h} + \sigma_e^{\ell h}$$

where δ acts as a risk premium and d is a behavioural parameter:

- $d = 0$ the trader is a fundamentalist;
- $d > 0$ the trader is a trend-chaser;
- $d < 0$ the trader is a trend-contrarian.

Some antecedents - Anufriev et al (2006)

A similar setup has been already investigated by a number of works:

Anufriev, Bottazzi, Pancotto (2006)

- 1 risky and 1 riskless asset;
- analysis restricted to the *deterministic skeleton*;
- yield realizations e_t are replaced by their mean value $\bar{e}$;
- studying the stable equilibria of this new system they identify a necessary condition requiring the survivor agent(s) to be *"the most aggressive one, i.e. the one who, among all agents, invests the highest wealth share in the risky asset"*.

$$\gamma_n \downarrow \quad \Longrightarrow \quad x_n \uparrow \quad \Longrightarrow \quad \varphi_n \uparrow$$

Some antecedents - Anufriev et al (2011)

Anufriev, Bottazzi, Marsili, Pin (2011)

- L risky and 1 riskless asset;
- no short positions: $x^{\ell}_{n,t} \in (0,1)$ and $\sum_{\ell} x^{\ell}_{n,t} < 1$;
- adopt the concept of *Procedurally Consistent equilibria*;
- impose both portfolio choices and wealth shares of every agent to remain constant over time, i.e. $\mathbf{x}_{n,t} = \overset{*}{\mathbf{x}}_n$ and $\varphi_{n,t} = \overset{*}{\varphi}_n$;
- they find the equilibrium variance-covariance matrix of returns:

$$\overset{*}{C} = \Sigma + \frac{1}{(1-\overset{*}{x})^2}\, \overset{*}{\mathbf{x}}{}' \Sigma\, \overset{*}{\mathbf{x}}\, \mathbf{1} \otimes \mathbf{1} + \frac{1}{1-\overset{*}{x}} \left(\Sigma\, \overset{*}{\mathbf{x}} \otimes \mathbf{1} + \mathbf{1} \otimes \Sigma\, \overset{*}{\mathbf{x}} \right)$$

  where $\overset{*}{x} = \sum_{\ell} \overset{*}{x}{}^{\ell}$ and $\otimes$ denotes the tensor product;
- The more the market is exposed towards risky security, i.e. $\overset{*}{x} \sim 1$, the higher the excess (co)variance $\overset{*}{C} - \Sigma$.

Some antecedents - Drawbacks

Although these approaches are indeed very insightful, they nonetheless exhibit some drawbacks:

- If applied to the L-risky-asset case, the deterministic skeleton yields fixed points characterized by constant returns of different assets and constant investment shares of different agents. It is therefore impossible to capture any time- and cross-correlation in the returns.
- The PCE approximates the dynamics of the system only when traders are strongly backward-looking in forming expectations, i.e. $\lambda \sim 0$. It is unusable when traders are more *naïve* (already with $\lambda = 0.15$).
- They both focus on equilibrium dynamics only.
- In equilibrium, all survivors behave identically, although they need not be homogeneous.
- An economy with randomly defined investment functions has probability zero of displaying any equilibrium with multiple survivors.

Our approach

We wish to perform:

- a numerical simulation
- imposing none of the previous restrictions
- allowing for a dynamic population
- introducing the possibility of a micro-failure

In such a framework we are able to capture the dynamics of the system as it is repeatedly perturbed away from the convergence to an equilibrium by the continuous entrance of new heterogeneous traders.

The approach resembles the LLS Microscopic Simulation model (Levy et al, 1994, 1995), although they make different assumptions: homogeneity in risk aversion; costant dividend; noisy investment functions by means of a random shock.

Our approach

Initialisation:

- N traders, heterogeneous in behavioural parameters $(\gamma, \lambda, d, \delta)$
- endowed with level of wealth $W_{n,0} > 0$ and information set $\mathcal{I}_0$
- L risky assets, with initial prevailing prices $\mathbf{p}_0$, paying dividends $\mathbf{d}_0$
- a dividend yield process with mean $\mathbf{e}$ and var-cov matrix Σ
- a bond with unitary price, yielding riskless return r_f

As time goes by:

- the decision-investment-pricing-dividend processes take place
- traders whose wealth share $\varphi_{n,t}$ goes (significantly close) to zero exit the market
- every τ time steps a new trader enters the market with random behavioural parameters and wealth endowment
- every ν time steps a randomly selected trader is forced to sell his entire portfolio with probability $\alpha > 0$

Our approach

The intuition runs as follows:

- the market continuously selects traders according to their investment functions;
- for a given level of expected returns, least risk averse traders on average see their wealth shares soar, and viceversa;
- the economy slowly converges to a single survivor equilibrium, the one including the least risk-averse trader;
- meanwhile, more risk averse traders are wiped out and new entrants face the survivors with randomly selected attitude;
- an entrant with high RA coefficient is quickly dominated by incumbents;
- if an entrant has the lowest RA coefficient, the dynamics start to converge towards a new equilibrium with this entrant as unique survivor;

Our approach

. . . continued:

- as time passes, the average market portfolio is increasingly exposed towards risky assets: $x \to 1$;
- the higher x_t, the higher the realized returns and their covariances;
- the price dynamics no longer follows the fundamental yield process;
- when the micro-failure strikes, a previously aggressive portfolio is nullified: $x_n = 0$, with n randomly drawn;
- under certain conditions, this occurrence is likely to trigger disruptive effects on the price returns dynamics.

A step further - Financial innovation

- A trader n can issue a new composite security A^{L+1} reproducing his own portfolio of risky assets and paying the average dividend yield

$$e_t^{L+1} = \sum_{\ell=1}^{L} x_{n,t}^{\ell} \cdot e_t^{\ell}$$

- The liquidity raised by selling the derivative can in principle be used by n for additional investment with the hope of reaping yield differentials and capital gains.
- But if once happens that the issuer is unable to pay the dividend then he must liquidate his portfolio, i.e. a micro-failure similar to the one assumed above.
- The introduction of the new security is likely to increase the speed/strenght of the trend towards riskier portfolios.