An agent-based model of intra-day financial markets dynamics

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Outline

1. Context and motivation
2. Stylised facts
3. The model
4. Simulations
5. Concluding remarks
during recent years, the availability of advanced technology has been substantially reducing the latency required to operate on financial markets, fostering market activity at increasingly higher frequencies

**Cont (2011)** time to execution dropped 25-fold between 2000 and 2010

**Carrion (2013)** 68.3% of NASDAQ dollar turnover attributable to HFT

**Aldridge (2013)** the majority of HFTs delivered positive returns in 2008, whereas 70% of LFTs lost money

### high-frequency traders
- high # of trades per day
- low average gain per trade
- low overnight inventories

### pros? cons?
- market quality
- volatility
- flash-crash
### Stylised facts

#### low- or cross-frequency
- properties of returns
- properties of volumes

#### high-frequency
- properties of timing and order-flow

#### agent-based models
- no model has yet addressed the high-frequency set of stylised facts
- difficulty in mapping simulation time into calendar time
some of the stylised facts have been already (singularly) investigated and linked to patterns of information diffusion

**our proposal**

- parsimonious financial agent-based model
- intra-day financial dynamics
- no role for information diffusion
- most of the stylised facts *jointly* emerge from the endogenous interaction of heterogeneous traders
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1. Context and motivation
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**SF1 leptokurtosis** unconditional distribution displays heavier tail w.r.t. Gaussian distribution

**SF2 no linear autocorrelation** positive autocorr. quickly fading away

**SF3 volatility clustering** positive autocorr. of absolute/squared value slowly fading away

**SF4 leverage effect** volatility is higher during price drops than during price surges $\rightarrow$ negative correlation between volatility (absolute returns) and returns
SF5 # price changes per day 10,000+ for blue-chips in liquid markets

SF6 autocorrelation of durations time intervals between subsequent trades are positively autocorrelated

SF7 fat-tailed durations distribution of durations displays a heavier tail w.r.t. exponential distribution

SF8 order-flow clustering buy orders tend to follow buy orders and sell orders tend to follow sell orders
**SF9 Volumes Autocorrelation** quantities exchanged in successive trades exhibit positive autocorrelation

**SF10 Volume/Volatility Correlation** (self-explanatory)

**SF11 U-shaped Activity** volumes peak during early morning and late afternoon
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Ingredients

- order-driven financial market
  - price-time priority
- single long-lived security
- no dividend
- no fundamental news
- $N$ heterogeneous agents
  - fundamentalists mean-reverters
  - chartists trend-followers and contrarians
- no strategy switching

- strict global schedule $\implies$ EURONEXT
- endogenous participation based on past volatility
- automatic order cancellation
Trader behaviour

\[
\hat{r}_{i,t+h}^F = w_i^F \cdot \log \left( \frac{p_{t+h}^F}{p_t} \right) + \varepsilon_t \quad \text{(fundamentalist)}
\]

\[
\hat{r}_{i,t+h}^C = w_i^C \cdot \log \left( \frac{p_t}{p_{t-h}} \right) + \varepsilon_t \quad \text{(chartist)}
\]

- fundamentalist sensitivity \( w_i^F \sim |\mathcal{N}(0, \sigma_F^2)| \)
- chartist sensitivity \( w_i^C \sim \mathcal{N}(\mu_C, \sigma_C^2) \)
- fundamental price \( p^F > 0 \)
- memory/horizon \( h \in \mathbb{N}_+ \)
- common i.i.d. noise \( \varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon^2) \)
Limit order

**definition**
a limit order submitted by trader $i$ at time $t$ is a triple

$$\ell_{i,t} = \{\text{price, quantity, validity}\}$$

$$= \{\text{round}(p_t \cdot \exp(\hat{r}_{i,t+h}), \text{tick}), \text{sgn}(\hat{r}_{i,t+h}), t + h\}$$

- $\text{round}(\cdot)$ is the rounding function
- $\text{tick}$ is the minimum price increment/decrement
- $\text{sgn}(\cdot)$ is the sign function

- no feedback from current time of the day
Order cancellation

Automatic cancellation

A stored order $\ell_{i,t}$ is automatically deleted from the book

- at its expiration time $t + h$
- if $i$ submits a new order with different sign (side)
- if $i$ submits a new order and $\ell_{i,t}$ is deemed unfavourable
  - new buy order at lower price
  - new sell order at higher price
Trader participation

**uniform activation**

- exactly one trader is activated at each time step, randomly selected from the population $N$

**endogenous activation**

- trader $i$ is active at time $t$ if

  $$|r_{\tau}| > \delta_{i,t} \sim |\mathcal{N}(0, \sigma_\delta^2)|$$

  where $\tau < t$ denotes the last time a trade occurred

- if $|r_{\tau}| < \delta_{i,t}, \forall i = 1, \ldots, N$ then uniform activation with probability $\phi > 0$
### Timing (Euronext)

<table>
<thead>
<tr>
<th>time</th>
<th>phase</th>
<th>duration</th>
</tr>
</thead>
<tbody>
<tr>
<td>from 7:15am to 9:00am</td>
<td>pre-opening</td>
<td>6,300 s</td>
</tr>
<tr>
<td>at 9:00am</td>
<td>opening auction</td>
<td>—</td>
</tr>
<tr>
<td>from 9:00am to 5:30pm</td>
<td>main trading session</td>
<td>30,600 s</td>
</tr>
<tr>
<td>from 5:30pm to 5:35pm</td>
<td>pre-closing</td>
<td>300 s</td>
</tr>
<tr>
<td>at 5:35pm</td>
<td>closing auction</td>
<td>—</td>
</tr>
<tr>
<td>10 hours, 20 minutes</td>
<td>—</td>
<td>37,200 s</td>
</tr>
</tbody>
</table>

1 simulation step $\iff$ 1 calendar second
Workflow

trader $i$ at time $t$

- active
  - automatic cancellation
  - accumulation phase
  - inactive

- form expectation
  - submit limit order
  - $\exists$ crossing order
    - trade occurs
    - $\forall$ crossing order
      - quantity exchanged
      - (new) price announced

- $\forall$ crossing order
  - store on order book

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we simulate the model numerically under three scenarios

**NT** : only noise traders

**FC** : fundamentalists and chartists with uniform participation

**EA** : fundamentalists and chartists with endogenous participation

<table>
<thead>
<tr>
<th>parameter</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>1,000</td>
</tr>
<tr>
<td>$p^F$</td>
<td>100</td>
</tr>
<tr>
<td>$\text{tick}$</td>
<td>0.001</td>
</tr>
<tr>
<td>$h$</td>
<td>1,000</td>
</tr>
<tr>
<td>$p_0$</td>
<td>$p^F$</td>
</tr>
</tbody>
</table>

at the beginning of the simulation all chartists are provided a history of past prices between $t = -h$ and $t = 0$ that evolves (backwards) as a pure random walk

- irregular time series are pooled into minute-by-minute data
- results are averaged across 100 Montecarlo simulations
NT scenario

avg. # of trades = 14,958  \( \kappa \approx 3.17 \)
Empirical quantiles
ACF Abs. Returns
ACF Durations
ACF Returns
ACF Volume

Time
Price

Lag
ACF Durations

0
20
40
60

Lag
Empirical quantile

Theoretical quantiles

Lag
ACF Volume

Lag
ACF Order-flow

Lag
ACF Abs. Returns

Time
Return

ACF Returns

−0.05
0.1
0.3

Lag
ACF Abs. Returns

−0.0015
0.0010

Lag
ACF Order-flow

−0.00010
0.00015

Lag
Leverage effect

avg. # of trades = 14,953 κ ≈ 14.5

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EA scenario

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avg. # of trades = 9,991

κ ≈ 13.96
### Stylised facts

<table>
<thead>
<tr>
<th>stylised fact</th>
<th>scenario</th>
</tr>
</thead>
<tbody>
<tr>
<td>SF1 leptokurtic returns</td>
<td>NT FC EA</td>
</tr>
<tr>
<td>SF2 no linear autocorr.</td>
<td>NT FC EA</td>
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<td>SF3 volatility clustering</td>
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<td>SF4 leverage effect</td>
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<td>SF5 # price changes</td>
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<td>SF6 autocorr. durations</td>
<td>NT FC EA</td>
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<td>SF7 fat-tailed durations</td>
<td>NT FC EA</td>
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<tr>
<td>SF8 order-flow clustering</td>
<td>NT FC EA</td>
</tr>
<tr>
<td>SF9 autocorr. volumes</td>
<td>NT FC EA</td>
</tr>
<tr>
<td>SF10 volume/volatility corr.</td>
<td>NT FC EA</td>
</tr>
<tr>
<td>SF11 U-shaped activity</td>
<td>NT FC EA</td>
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## Recap

### Assumptions
- No information diffusion – everything is common knowledge
- Trading emerges as the consequence of differing (stable) beliefs
  - Fundamentalists vs. chartists
- Strict timing and microstructure from Euronext
- Endogenous participation based on past volatility

### Results

<table>
<thead>
<tr>
<th>NT</th>
<th>Slight dependence in returns quickly fading, # trades/day</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC</td>
<td>$[NT] \oplus$ leptokurtosis and volatility clustering</td>
</tr>
<tr>
<td>EA</td>
<td>$[FC] \oplus$ dependence in timing, volumes, and order-flow</td>
</tr>
</tbody>
</table>
Extensible extensions

- time feedback in trading strategies
- budget constraint/leverage requirement
- more complex chartist specification $\Rightarrow$ leverage effect
- calibration of model parameters $\Rightarrow$ policy experiments

$\Rightarrow$ U-shaped seasonality
Thank you very much!

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Dolfins

...and props to the European Project 640772 – DOLFINS – H2020-FETPROACT-2014 for financial support
### NT scenario

<table>
<thead>
<tr>
<th>param.</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\varepsilon_t)</td>
<td>(\mathcal{N}(0,5e-5))</td>
</tr>
<tr>
<td>(w_i^F)</td>
<td>0</td>
</tr>
<tr>
<td>(w_i^C)</td>
<td>0</td>
</tr>
<tr>
<td>(\delta_t)</td>
<td>(+\infty)</td>
</tr>
<tr>
<td>(\phi)</td>
<td>1</td>
</tr>
</tbody>
</table>
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</thead>
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<tr>
<td>$\epsilon_t$</td>
<td>$\mathcal{N}(0, 5e^{-5})$</td>
</tr>
<tr>
<td>$w_i^F$</td>
<td>$</td>
</tr>
<tr>
<td>$w_i^C$</td>
<td>$\mathcal{N}(0.01, 0.1)$</td>
</tr>
<tr>
<td>$\delta_t$</td>
<td>$+\infty$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$1$</td>
</tr>
</tbody>
</table>
EA scenario

<table>
<thead>
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</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_t$</td>
<td>$\mathcal{N}(0, 5e-5)$</td>
</tr>
<tr>
<td>$w^F_i$</td>
<td>$</td>
</tr>
<tr>
<td>$w^C_i$</td>
<td>$\mathcal{N}(0.01, 0.1)$</td>
</tr>
<tr>
<td>$\delta_t$</td>
<td>$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$1/3$</td>
</tr>
</tbody>
</table>
Figure: Price series (left) and return series (right) for a typical trading day
Figure: Autocorrelation of returns (left) and of absolute returns (right)
Figure: Autocorrelation of durations (left) and Q-Q of their distribution (right)
Figure: Autocorrelation of volumes (left) and of order-flow (right)
Figure: Volume/volatility correlation (left) and leverage effect (right)
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FC scenario (iii)

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