An agent-based model of intra-day financial markets dynamics

Jacopo Staccioli[†] Mauro Napoletano[‡]

[†] *Institute of Economics - Scuola Superiore Sant'Anna, Pisa*

[‡] *OFCE – Sciences Po, Sophia Antipolis*

This version: 30th June 2018

Outline

Context

during recent years, the availability of advanced technology has been substantially reducing the latency required to operate on financial markets, fostering market activity at increasingly higher frequencies

- **Cont (2011)** time to execution dropped 25-fold between 2000 and 2010
- **Carrion (2013)** 68.3% of NASDAQ dollar turnover attributable to HFT
- **Aldridge (2013)** the majority of HFTs delivered positive returns in 2008, whereas 70% of LFTs lost money

high-frequency traders

- high # of trades per day
- low average gain per trade
- low overnight inventories

pros? cons?

- market quality
- volatility
- flash-crash

Stylised facts

low- or cross-frequency

- properties of returns
- properties of volumes

high-frequency

- properties of timing and order-flow

agent-based models

- no model has yet addressed the high-frequency set of stylised facts
- difficulty in mapping simulation time into calendar time

Contribution

some of the stylised facts have been already (singularly) investigated and linked to patterns of information diffusion

our proposal

- parsimonious financial agent-based model
- intra-day financial dynamics
- no role for information diffusion
- most of the stylised facts *jointly* emerge from the endogenous interaction of heterogeneous traders

Outline

Returns

SF1 LEPTOKURTOSIS unconditional distribution displays heavier tail w.r.t. Gaussian distribution

SF2 NO LINEAR AUTOCORRELATION positive autocorr. quickly fading away

SF3 VOLATILITY CLUSTERING positive autocorr. of absolute/squared value slowly fading away

SF4 LEVERAGE EFFECT volatility is higher during price drops than during price surges $\Longrightarrow$ negative correlation between volatility (absolute returns) and returns

Timing

SF5 # PRICE CHANGES PER DAY 10,000+ for blue-chips in liquid markets

SF6 AUTOCORRELATION OF DURATIONS time intervals between subsequent trades are positively autocorrelated

SF7 FAT-TAILED DURATIONS distribution of durations displays a heavier tail w.r.t. exponential distribution

SF8 ORDER-FLOW CLUSTERING buy orders tend to follow buy orders and sell orders tend to follow sell orders

Volumes

SF9 VOLUMES AUTOCORRELATION quantities exchanged in successive trades exhibit positive autocorrelation

SF10 VOLUME/VOLATILITY CORRELATION (self-explanatory)

SF11 U-SHAPED ACTIVITY volumes peak during early morning and late afternoon

Outline

Ingredients

- order-driven financial market
  - price-time priority
- single long-lived security
- no dividend
- no fundamental news
- N heterogeneous agents

fundamentalists	mean-reverters
chartists	trend-followers and contrarians

- no strategy switching
- strict global schedule $\Longrightarrow$ EURONEXT
- endogenous participation based on past volatility
- automatic order cancellation

Trader behaviour

$$\hat{r}^F_{i,t+h} = w^F_i \cdot \log\left(\frac{p^F}{p_t}\right) + \varepsilon_t \qquad \text{(fundamentalist)}$$

$$\hat{r}^C_{i,t+h} = w^C_i \cdot \log\left(\frac{p_t}{p_{t-h}}\right) + \varepsilon_t \qquad \text{(chartist)}$$

- fundamentalist sensitivity $w^F_i \sim |\mathcal{N}(0,\ \sigma^2_F)|$
- chartist sensitivity $w^C_i \sim \mathcal{N}(\mu_C,\ \sigma^2_C)$
- fundamental price $p^F > 0$
- memory/horizon $h \in \mathbb{N}_+$
- common *i.i.d.* noise $\varepsilon_t \sim \mathcal{N}(0, \sigma^2_\varepsilon)$

Limit order

definition

a limit order submitted by trader i at time t is a triple

$$\ell_{i,t} = \{\, \texttt{price}, \ \texttt{quantity}, \ \texttt{validity} \,\}$$

$$= \Big\{ \operatorname{round}\Big(p_t \cdot \exp(\hat{r}_{i,t+h}), \ tick\Big), \quad \operatorname{sgn}(\hat{r}_{i,t+h}), \quad t+h \Big\}$$

- $\operatorname{round}(\cdot)$ is the rounding function
- *tick* is the minimum price increment/decrement
- $\operatorname{sgn}(\cdot)$ is the sign function

- no feedback from current time of the day

Order cancellation

automatic cancellation

a stored order $\ell_{i,t}$ is automatically deleted from the book

- at its expiration time $t + h$
- if i submits a new order with different sign (side)
- if i submits a new order and $\ell_{i,t}$ is deemed unfavourable
  - new buy order at lower price
  - new sell order at higher price

Trader participation

uniform activation

- exactly one trader is activated at each time step, randomly selected from the population N

endogenous activation

- trader i is active at time t if

$$|\, r_\tau \,| > \delta_{i,t} \sim |\, \mathcal{N}\,(0,\, \sigma_\delta^2\,)\,|$$

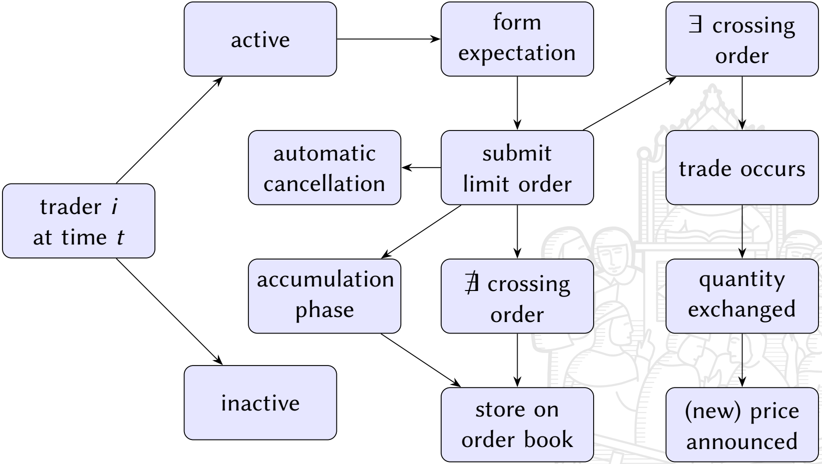
where $\tau < t$ denotes the last time a trade occurred
- if $|r_\tau| < \delta_{i,t},\ \forall i = 1, \ldots, N$ then uniform activation with probability $\phi > 0$

Timing (Euronext)

time	phase	duration
from 7:15am to 9:00am	pre-opening	6,300 s
at 9:00am	opening auction	—
from 9:00am to 5:30pm	main trading session	30,600 s
from 5:30pm to 5:35pm	pre-closing	300 s
at 5:35pm	closing auction	—
10 hours, 20 minutes	—	37,200 s

1 simulation step $\Longleftrightarrow$ 1 calendar second

Workflow



active
form expectation
∃ crossing order
automatic cancellation
submit limit order
trade occurs
trader i at time t
accumulation phase
∄ crossing order
quantity exchanged
inactive
store on order book
(new) price announced

Outline

we simulate the model numerically under three scenarios

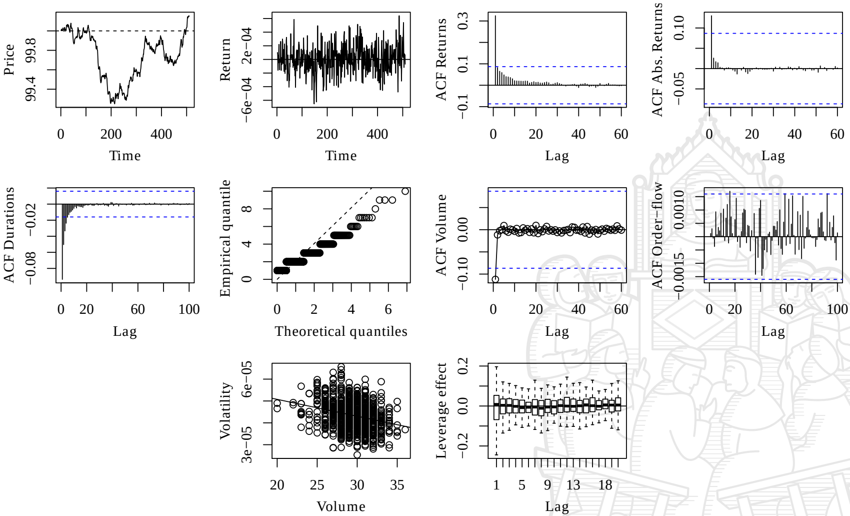
- NT : only noise traders
- FC : fundamentalists and chartists with uniform participation
- EA : fundamentalists and chartists with endogenous participation

parameter	**value**
N	$1,000$
p^F	100
tick	0.001
h	$1,000$
p_0	p^F

at the beginning of the simulation all chartists are provided a history of past prices between $t = -h$ and $t = 0$ that evolves (backwards) as a pure random walk

- irregular time series are pooled into minute-by-minute data
- results are averaged across 100 Montecarlo simulations

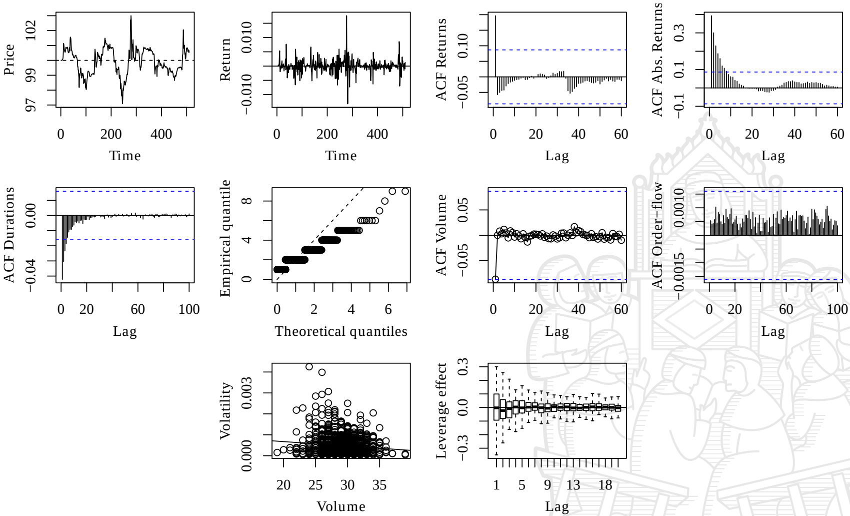
NT scenario



param

avg. # of trades $= 14,958$ $\kappa \approx 3.17$

zoom

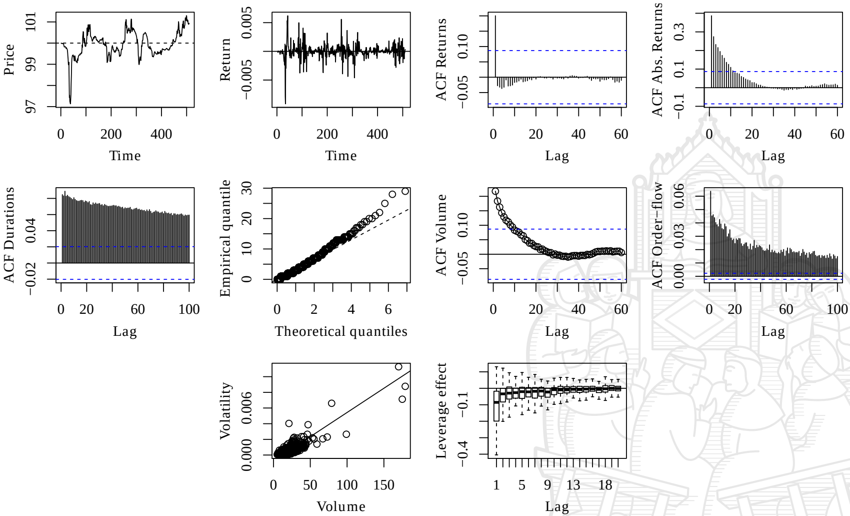


▸ param

avg. # of trades $= 14,953$ $\kappa \approx 14.5$

▸ zoom

EA scenario



param

avg. # of trades $= 9{,}991$ $\kappa \approx 13.96$

zoom

Stylised facts

stylised fact		scenario		
		NT	FC	EA
SF1	leptokurtic returns	✗	✓	✓
SF2	no linear autocorr.	✓	✓	✓
SF3	volatility clustering	✗	✓	✓
SF4	leverage effect	✗	✗	✗
SF5	# price changes	✓	✓	✓
SF6	autocorr. durations	✗	✗	✓
SF7	fat-tailed durations	✗	✗	✓
SF8	order-flow clustering	✗	✗	✓
SF9	autocorr. volumes	✗	✗	✓
SF10	volume/volatility corr.	✗	✗	✓
SF11	U-shaped activity	✗	✗	✗

Outline

Recap

assumptions

- no information diffusion – everything is common knowledge
- trading emerges as the consequence of differing (stable) beliefs
  - fundamentalists vs. chartists
- strict timing and microstructure from EURONEXT
- endogenous participation based on past volatility

results

NT	slight dependence in returns quickly fading, # trades/day
FC	[NT] ⊕ leptokurtosis and volatility clustering
EA	[FC] ⊕ dependence in timing, volumes, and order-flow

Extensions

conceivable extensions

- time feedback in trading strategies
- budget constraint/leverage requirement

$\Big\}\Rightarrow$ U-shaped seasonality

- more complex chartist specification $\Rightarrow$ leverage effect
- calibration of model parameters $\Rightarrow$ policy experiments

Thank you very much!

j.staccioli<at>santannapisa.it

Dolfins project

...and props to the European Project 640772 - DOLFINS - H2020-FETPROACT-2014 for financial support

NT scenario

param.	value
ε_t	$\mathcal{N}(0, 5e{-}5)$
w_i^F	0
w_i^C	0
δ_t	$+\infty$
ϕ	1

back

FC scenario

param.	value
ε_t	$\mathcal{N}(0, 5e{-}5)$
w_i^F	$\lvert\mathcal{N}(0, 0.001)\rvert$
w_i^C	$\mathcal{N}(0.01, 0.1)$
δ_t	$+\infty$
ϕ	1

back

EA scenario

param.	value
ε_t	$\mathcal{N}(0, 5e{-}5)$
w_i^F	$\lvert\mathcal{N}(0, 0.001)\rvert$
w_i^C	$\mathcal{N}(0.01, 0.1)$
δ_t	$\lvert\mathcal{N}(0, 0.3)\rvert$
ϕ	$1/3$

back

NT scenario (i)

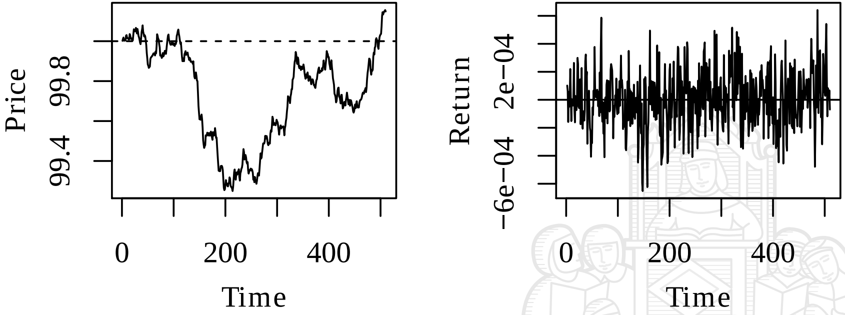


Figure: Price series (left) and return series (right) for a typical trading day

back

NT scenario (ii)

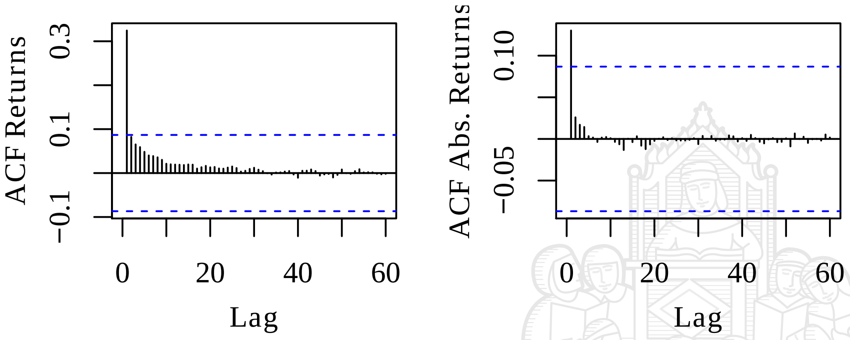


Figure: Autocorrelation of returns (left) and of absolute returns (right)

back

NT scenario (iii)

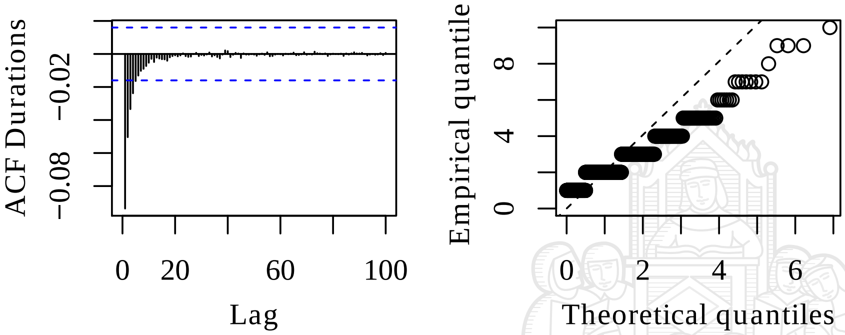


Figure: Autocorrelation of durations (left) and Q-Q of their distribution (right)

back

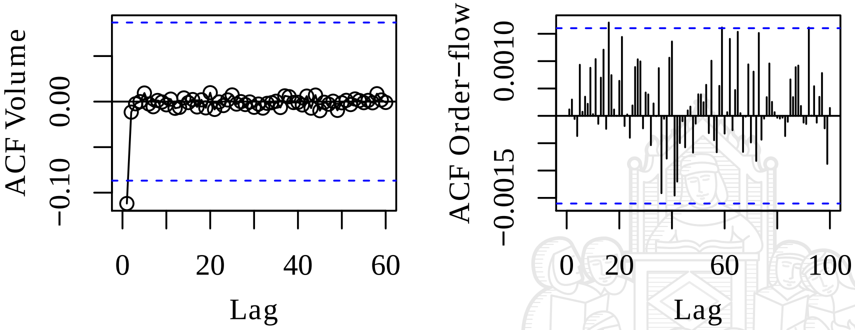


Figure: Autocorrelation of volumes (left) and of order-flow (right)

back

NT scenario (v)

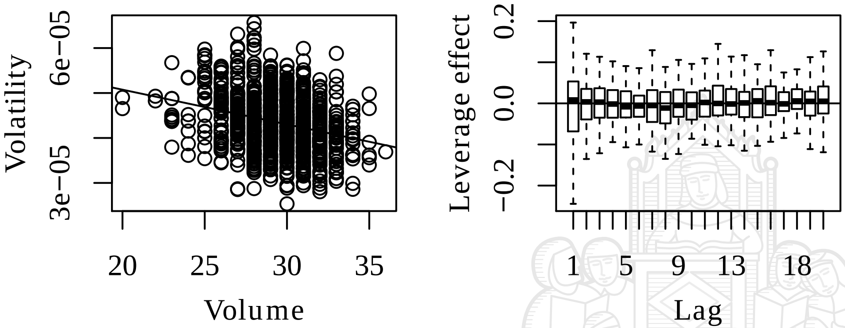


Figure: Volume/volatility correlation (left) and leverage effect (right)

back

FC scenario (i)

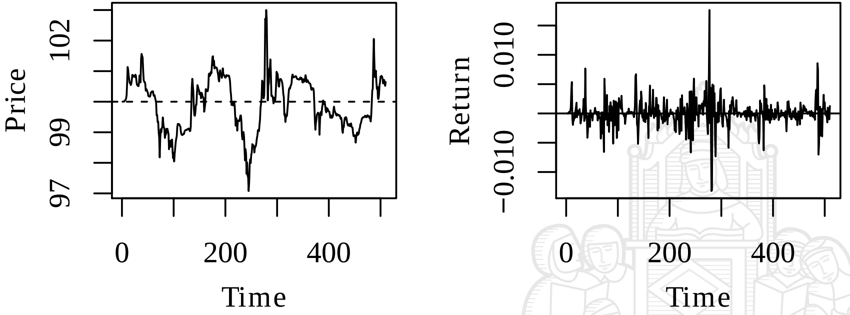


Figure: Price series (left) and return series (right) for a typical trading day

back

FC scenario (ii)

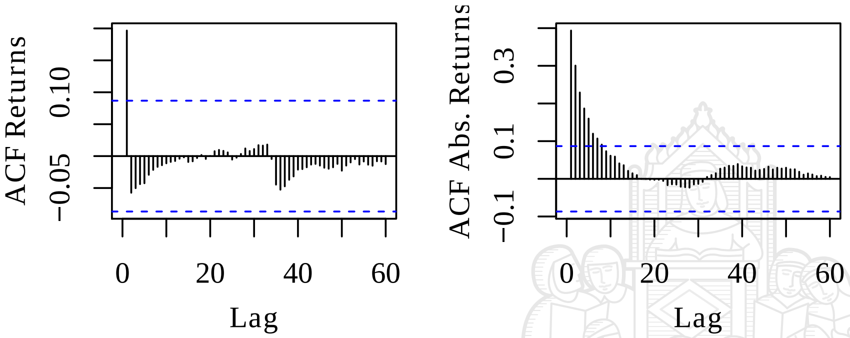
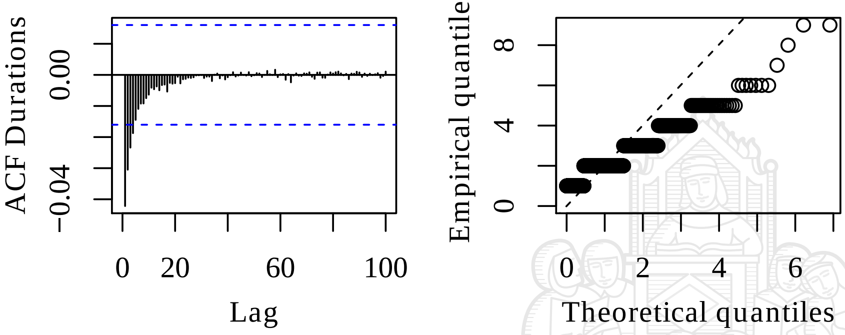


Figure: Autocorrelation of returns (left) and of absolute returns (right)

back

FC scenario (iii)



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FC scenario (iv)

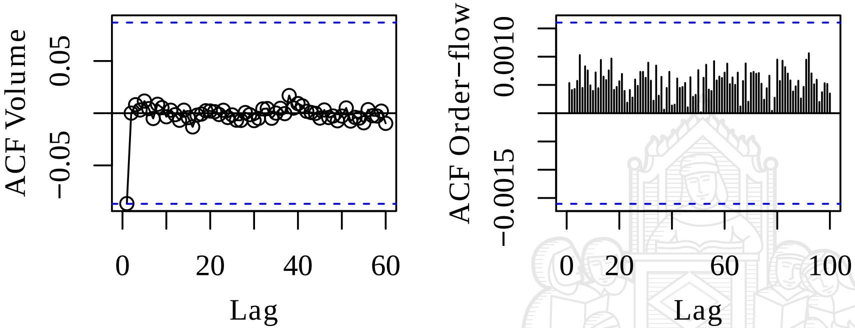

Figure: Autocorrelation of volumes (left) and of order-flow (right)

back

FC scenario (v)

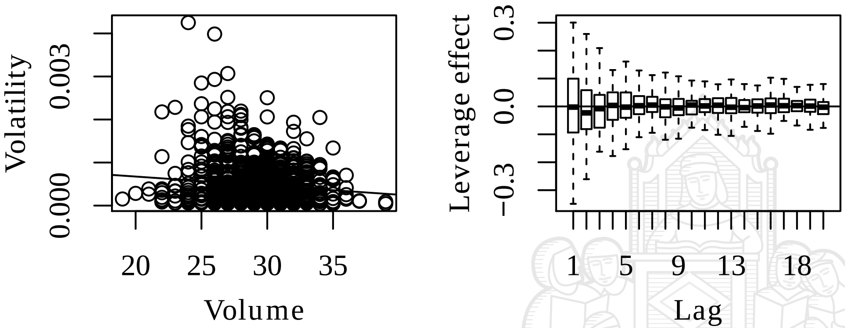

Figure: Volume/volatility correlation (left) and leverage effect (right)

back

EA scenario (i)

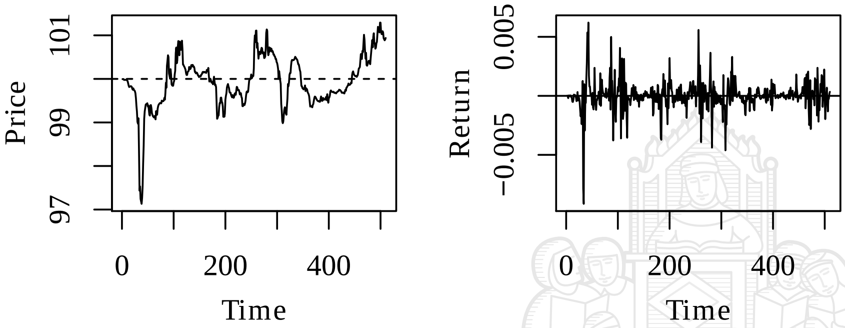

Figure: Price series (left) and return series (right) for a typical trading day

back

EA scenario (ii)

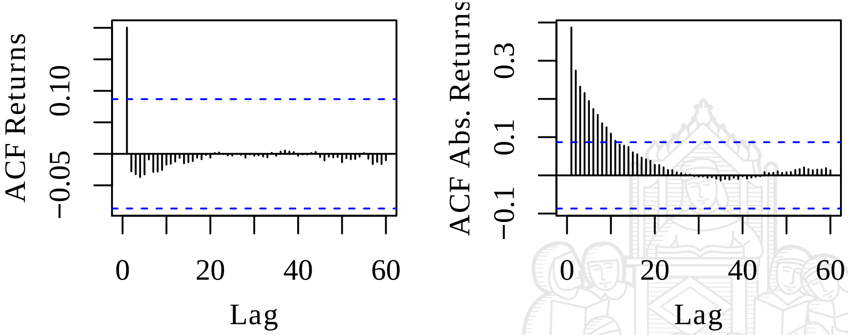

Figure: Autocorrelation of returns (left) and of absolute returns (right)

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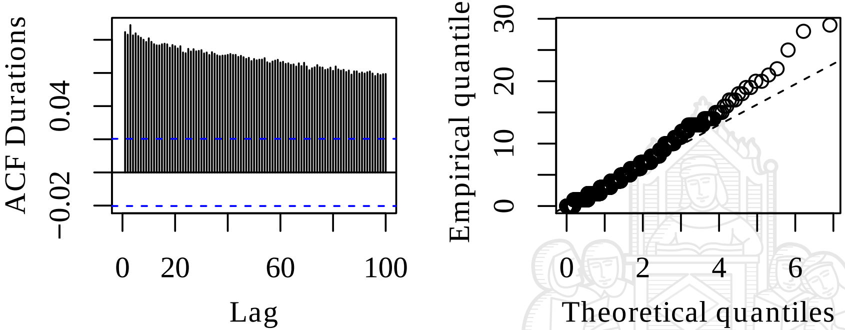

Figure: Autocorrelation of durations (left) and Q-Q of their distribution (right)

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EA scenario (iv)

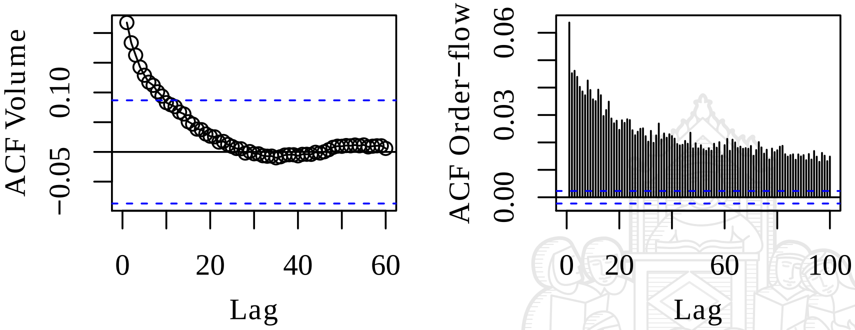
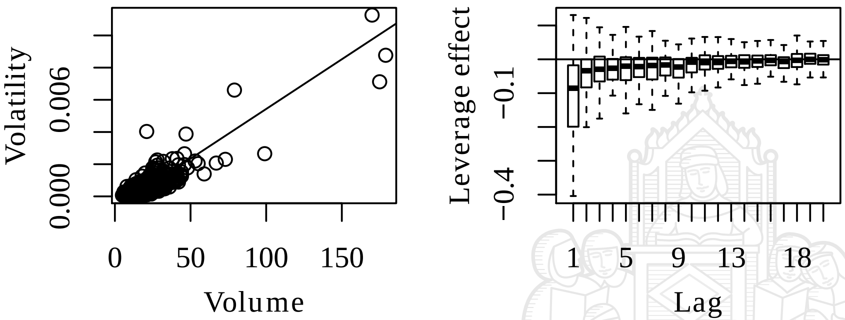


Figure: Autocorrelation of volumes (left) and of order-flow (right)

back

EA scenario (v)



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